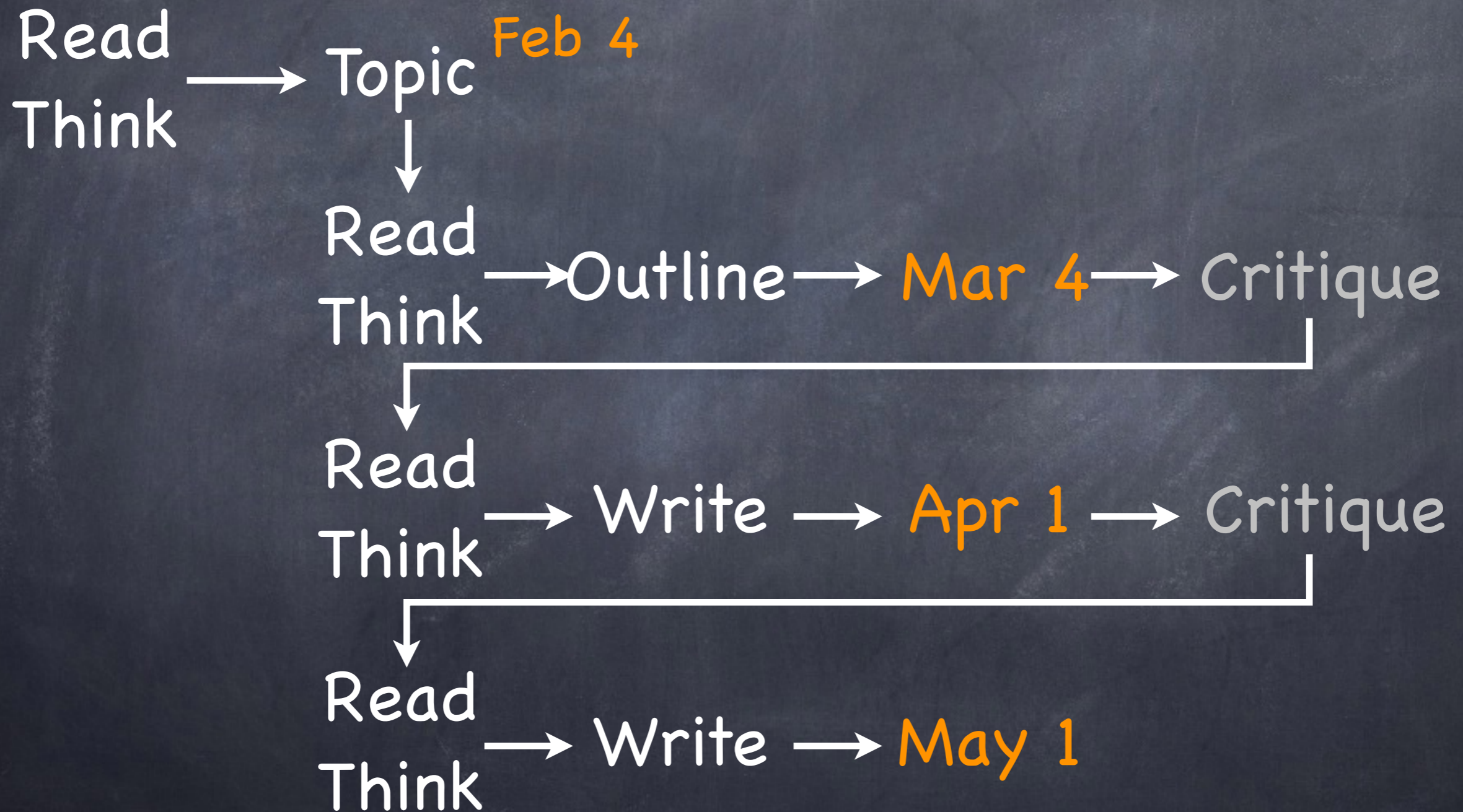


CSC242: Artificial Intelligence

Lecture 2: Problem Solving

Upper Level Writing



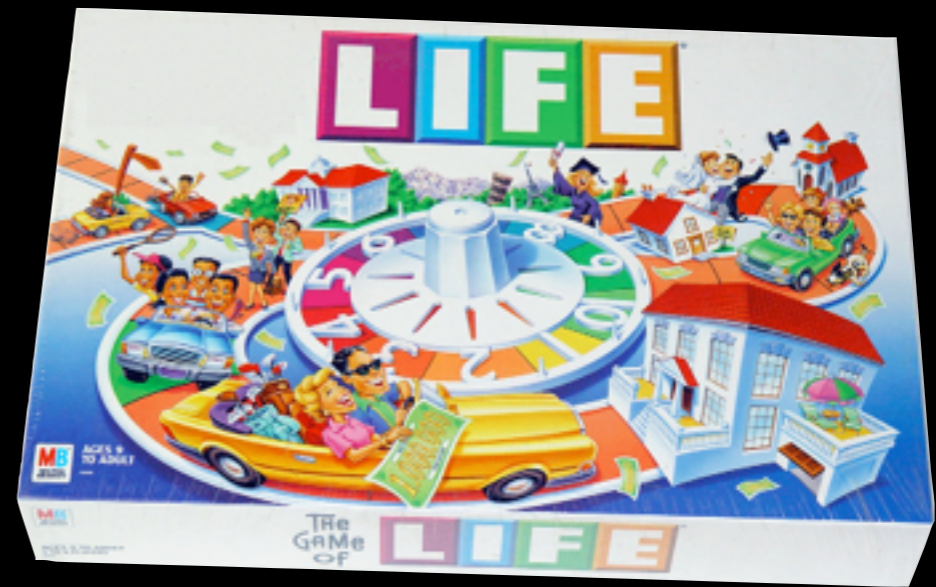
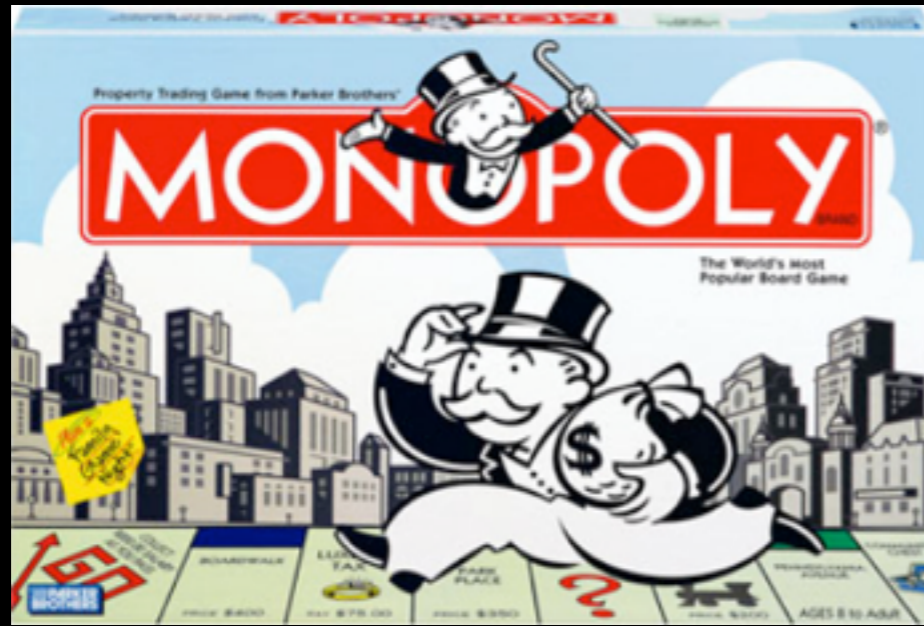
One More Policy

One More Policy

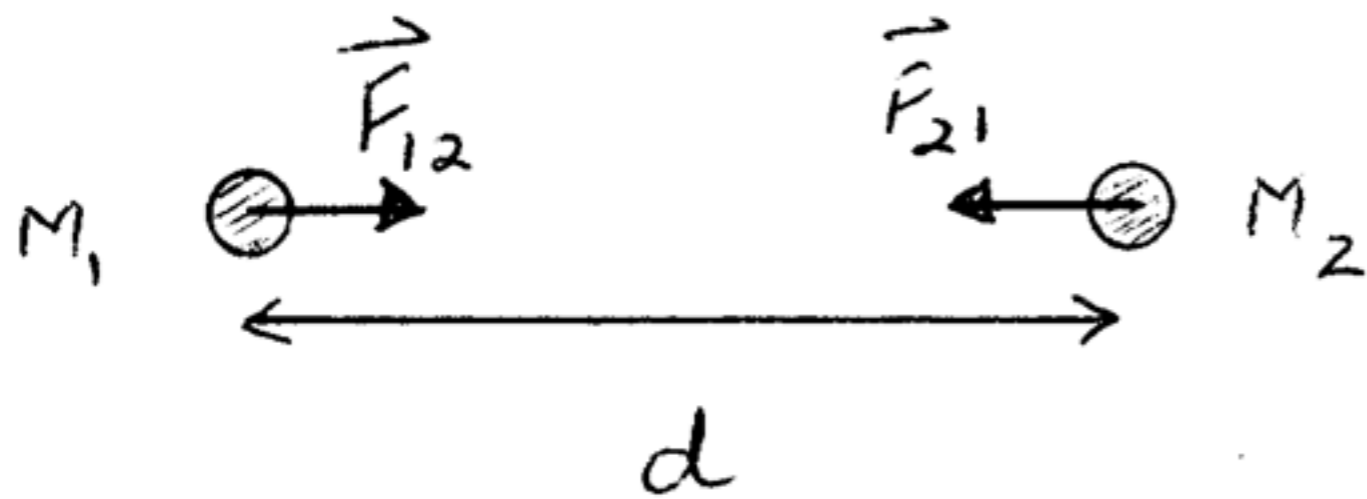


General Problem Solving









$$|\vec{F}_{12}| = |\vec{F}_{21}| = \frac{G M_1 M_2}{d^2}$$

Cooperative Problem Solving

Cooperative Problem Solving



Cooperative Problem Solving





CSC242

We don't make the computers.*
We make the computers solve problems.

*Or the programming languages, compilers, debuggers, databases, graphics pipelines, network protocols, web servers, ...

Our first problem

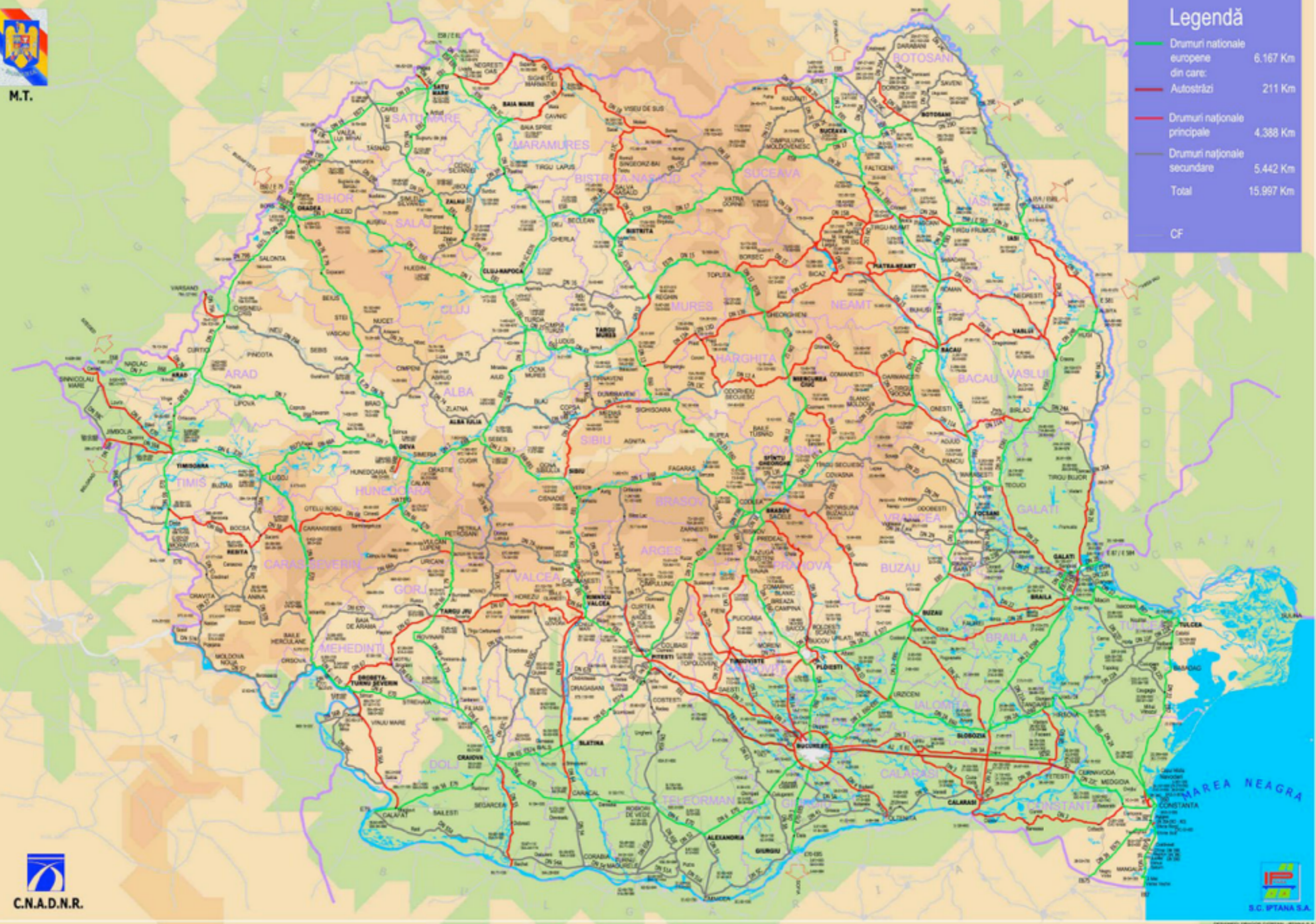
RETEAUA DRUMURILOR NATIONALE DIN ROMANIA



M.T.

Legendă

	Drumuri naționale europene	6.167 Km
	Autostrăzi	211 Km
	Drumuri naționale principale	4.388 Km
	Drumuri naționale secundare	5.442 Km
	Total	15.997 Km
	CF	



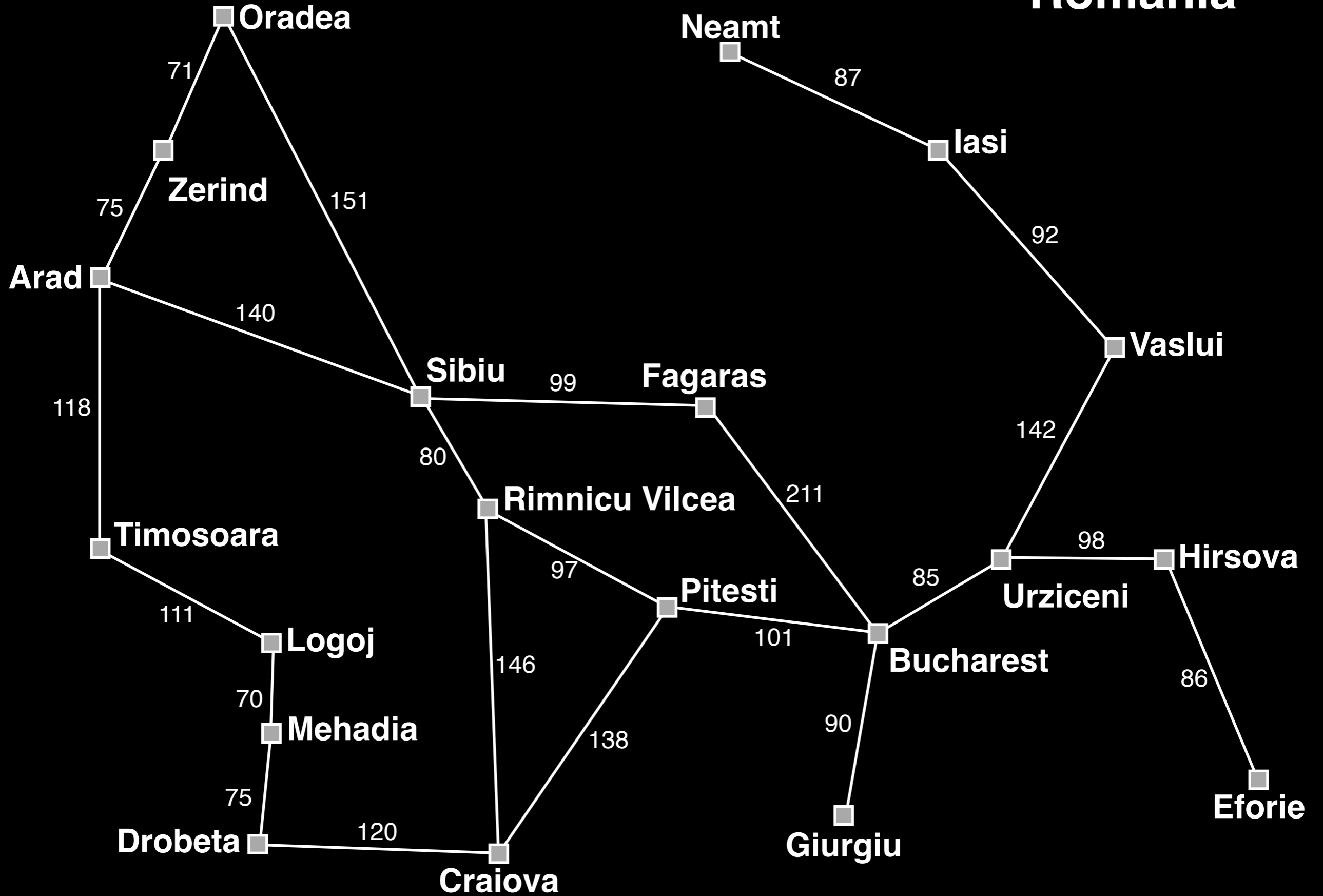
C.N.A.D.N.R.



S.C. IPTANA S.A.

DESIGN: DRAGO CORTAN - IPTANA S.A.

Romania



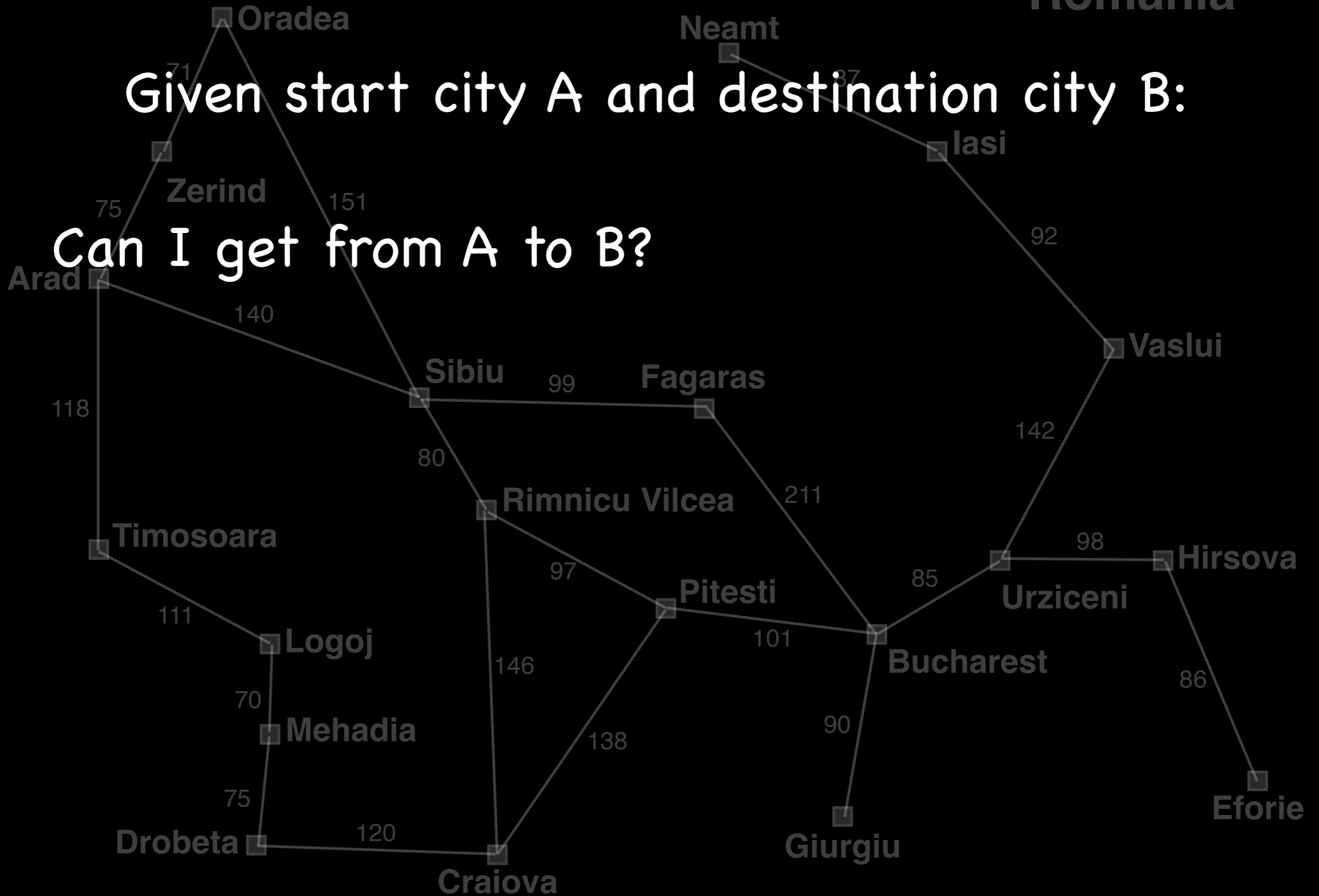
Romania



Romania

Given start city A and destination city B:

Can I get from A to B?

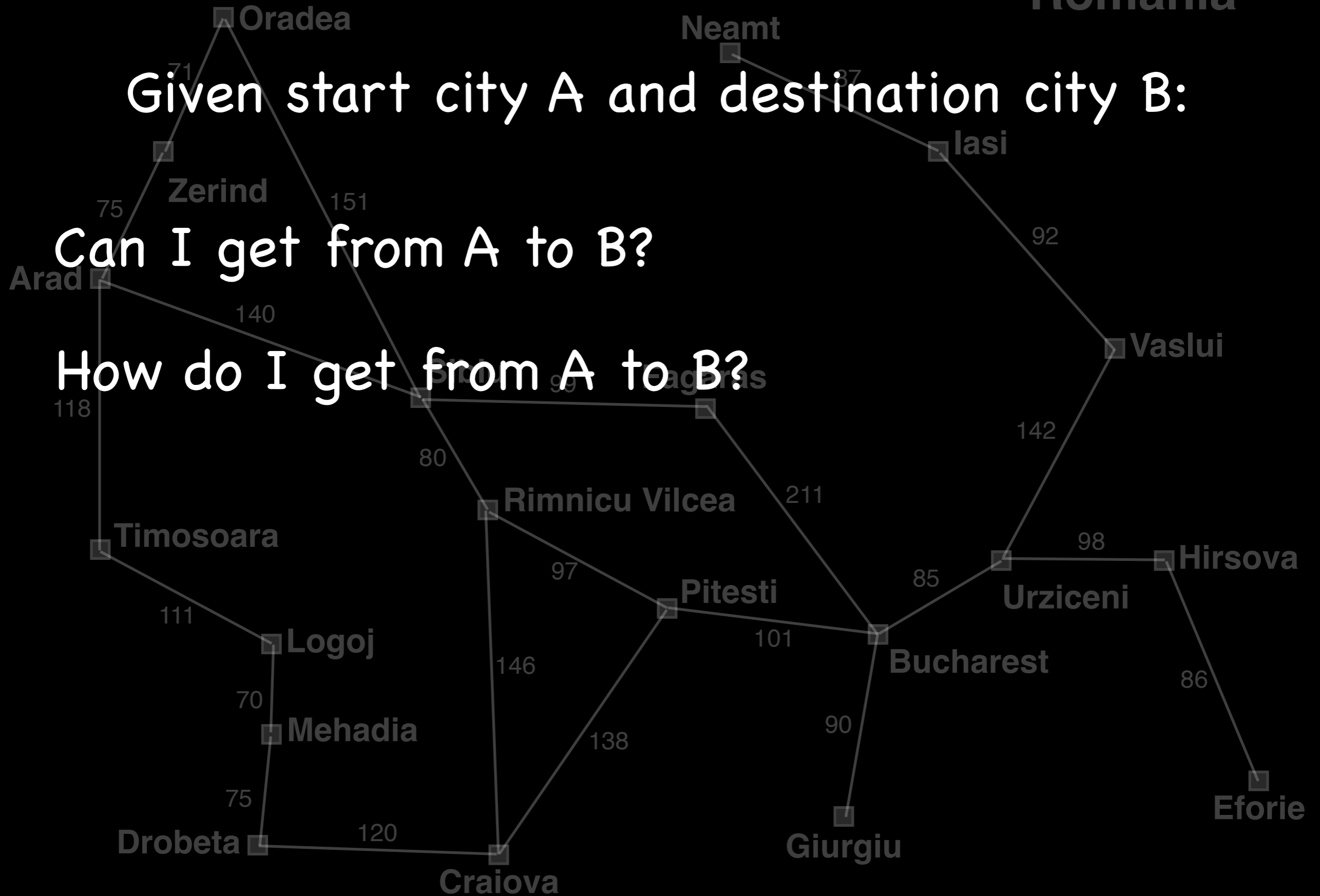


Romania

Given start city A and destination city B:

Can I get from A to B?

How do I get from A to B?

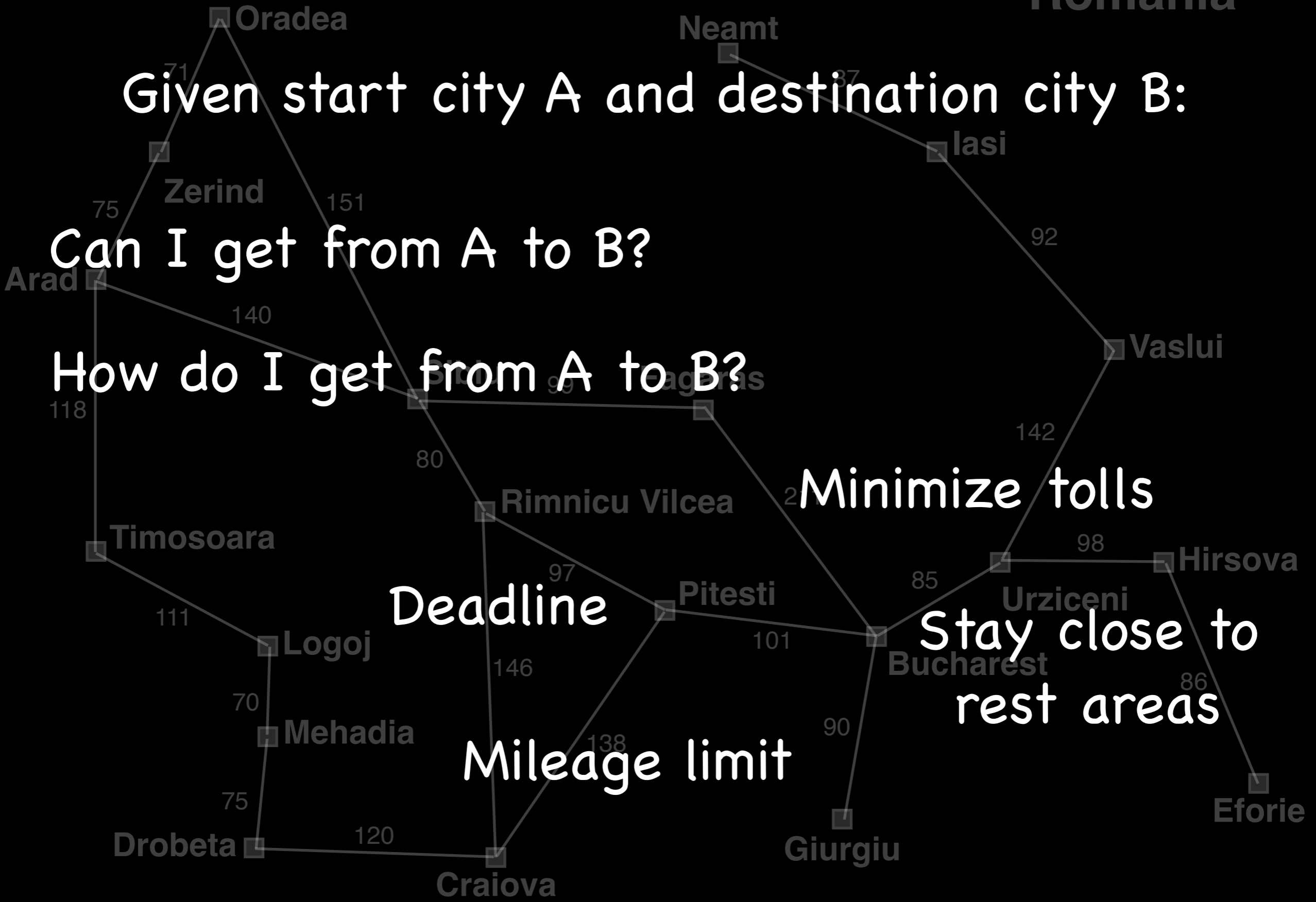


Romania

Given start city A and destination city B:

Can I get from A to B?

How do I get from A to B?



Minimize tolls

Deadline

Stay close to rest areas

Mileage limit

Romania

Cities



Romania

Cities

Roads: connect cities



Romania

Cities

Roads: connect cities

Distances between cities



Romania

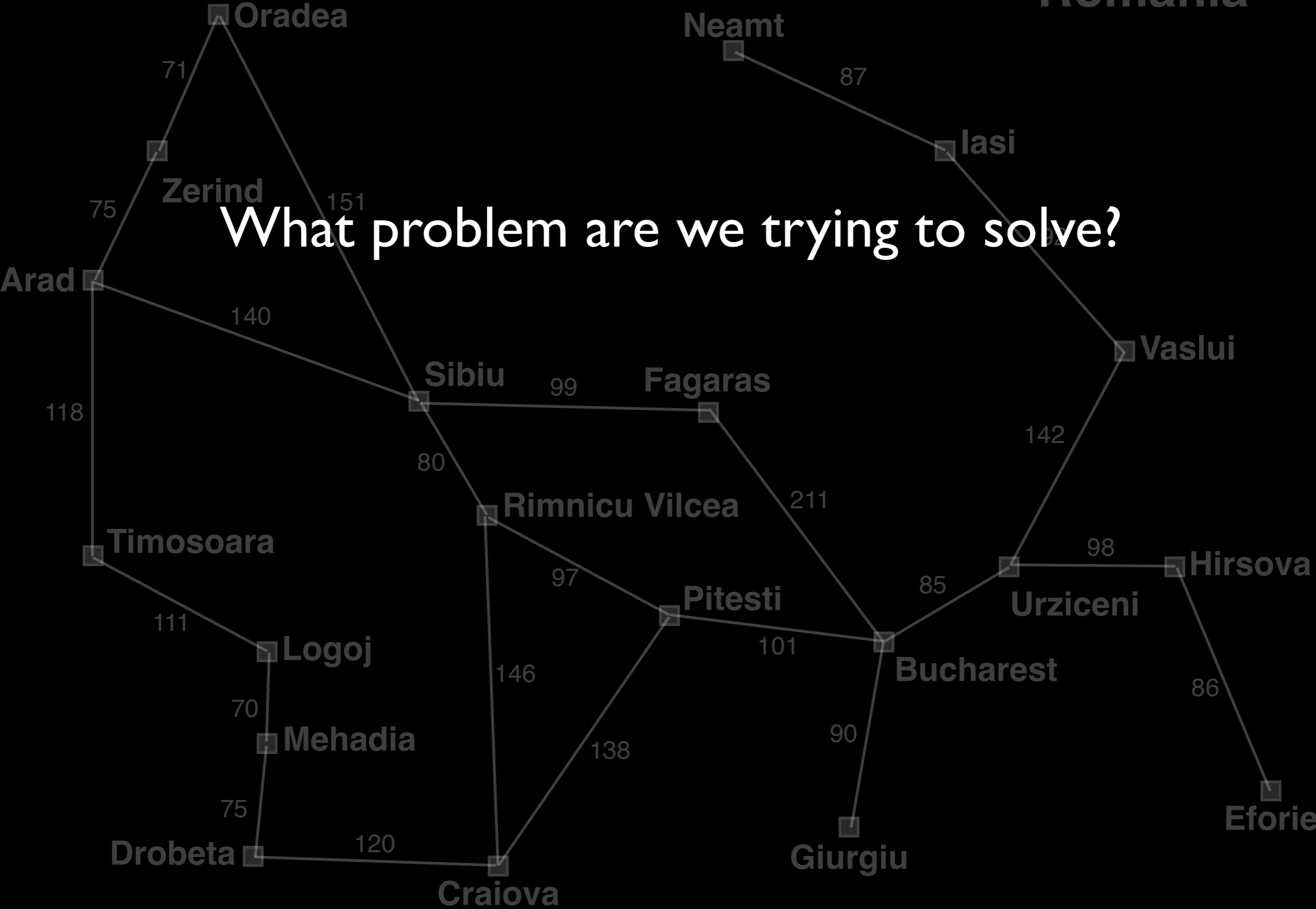


Romania

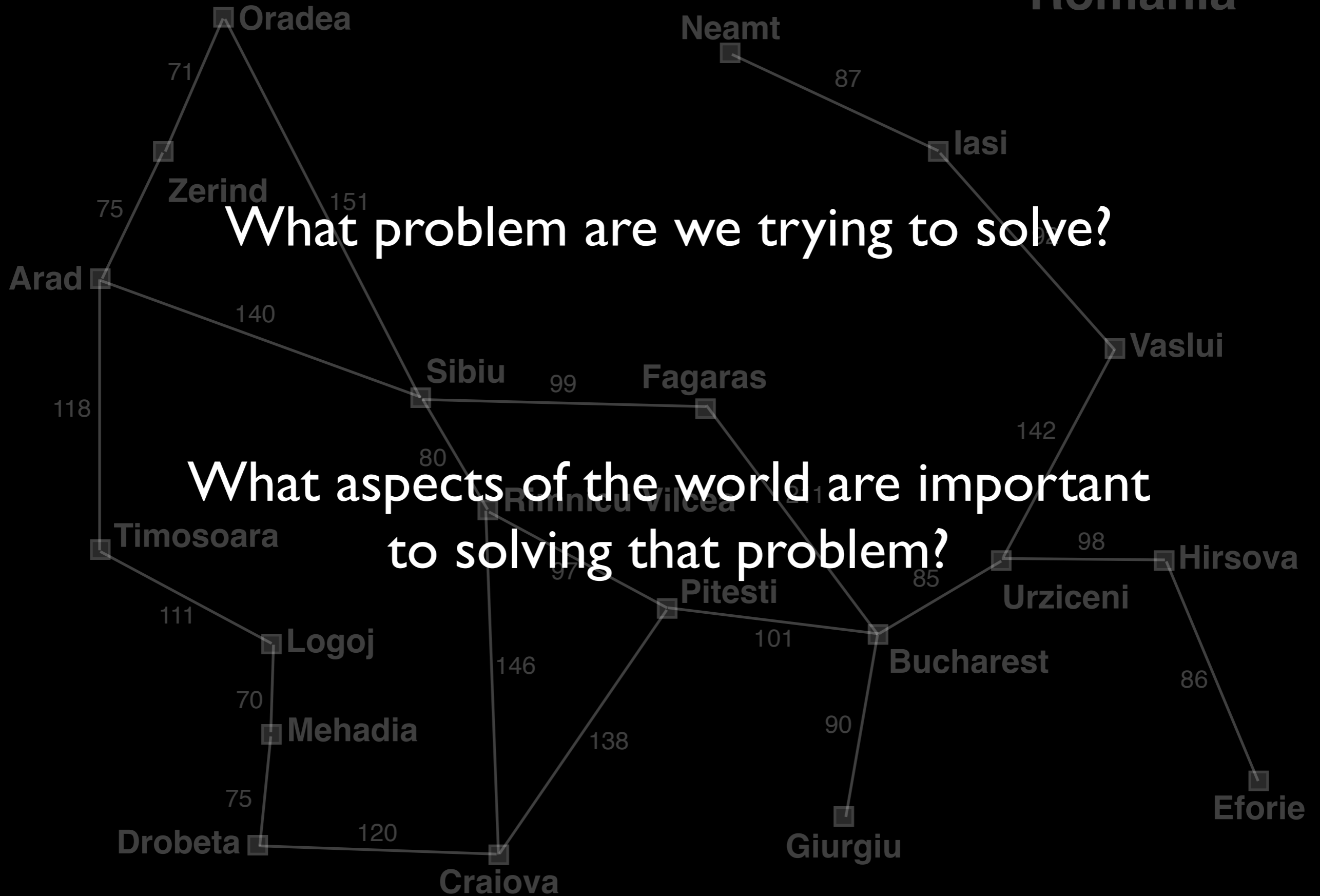


Romania

What problem are we trying to solve?



Romania

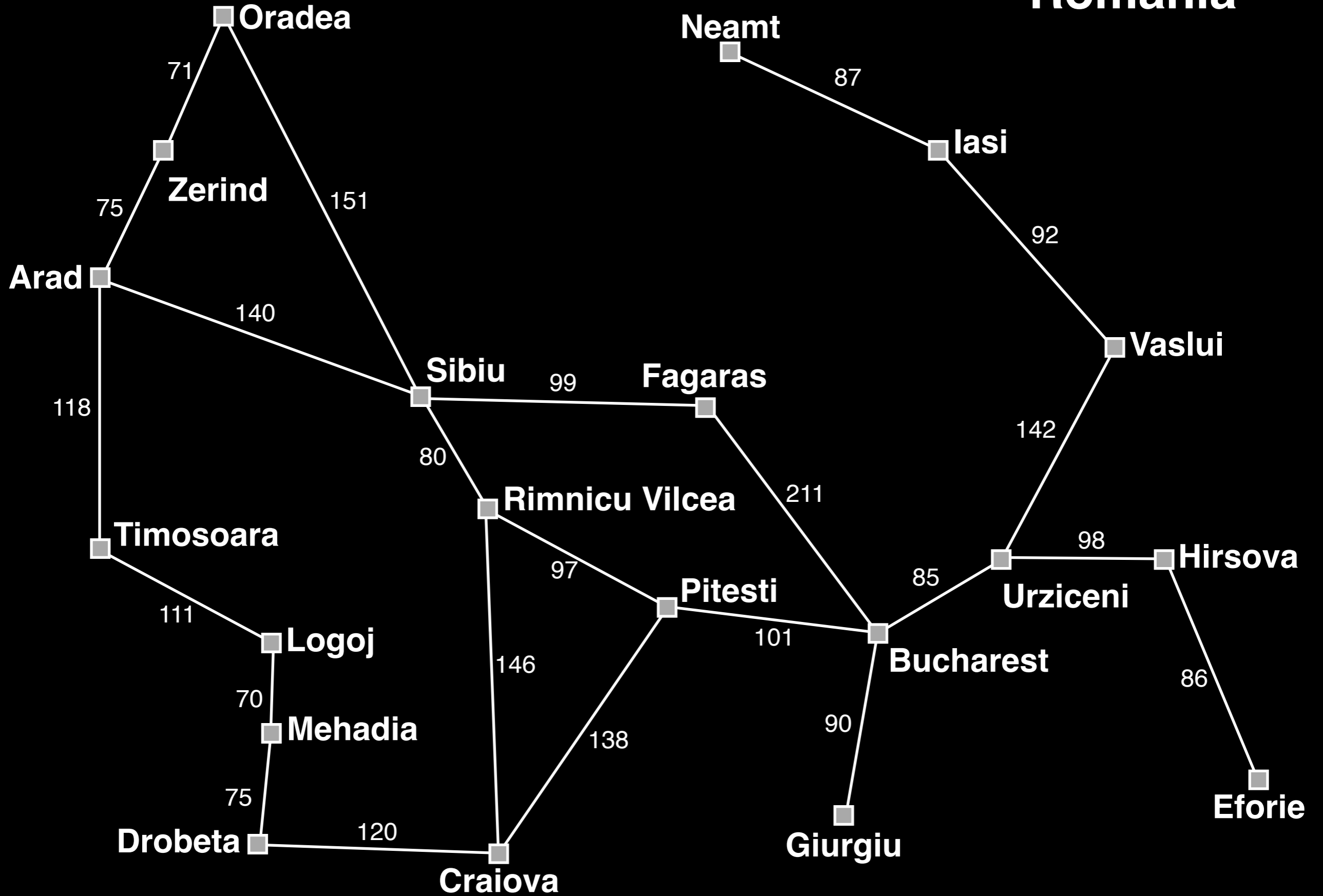


What problem are we trying to solve?

What aspects of the world are important to solving that problem?

Problem Solving by Computers

Romania



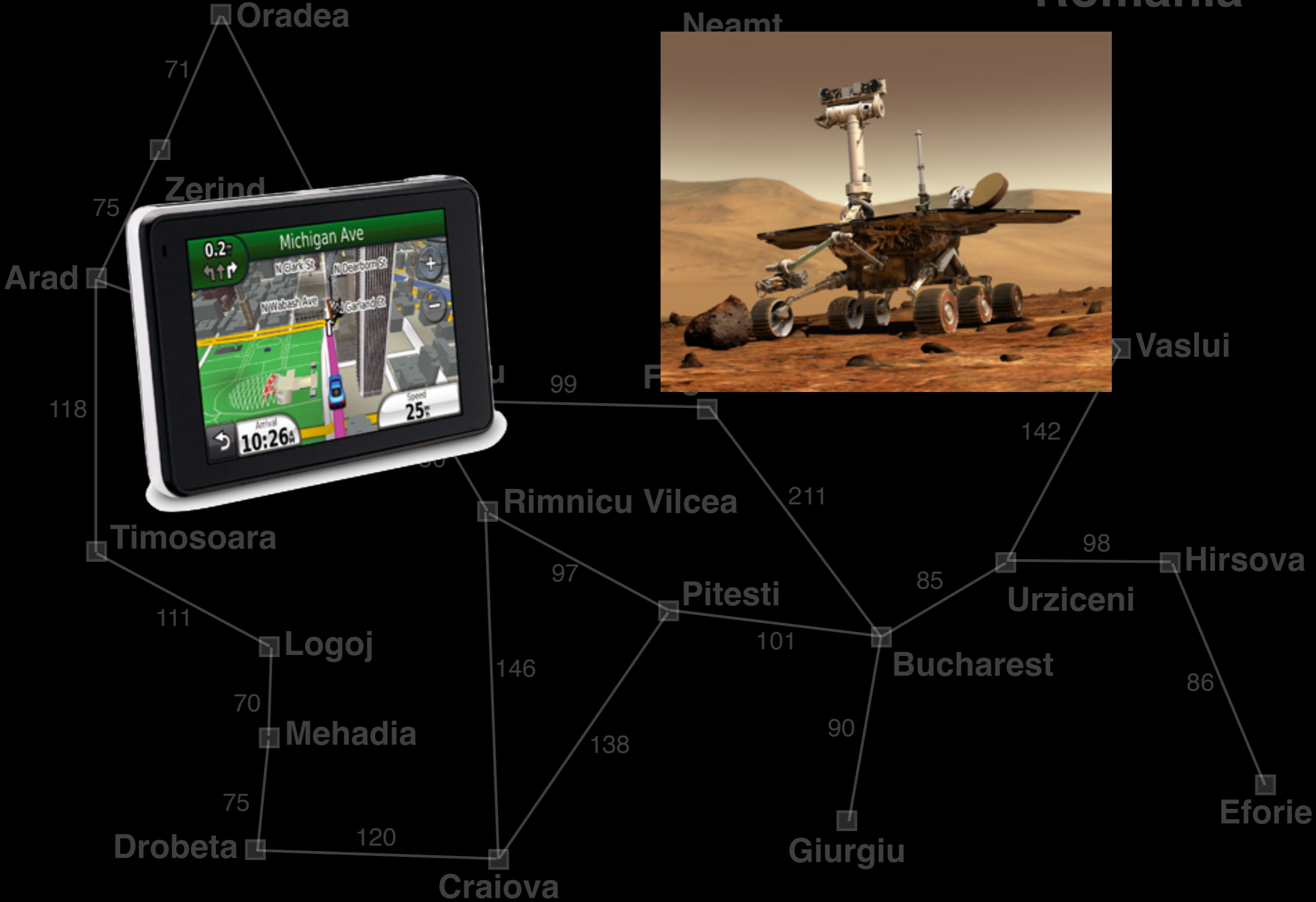
Romania



Romania



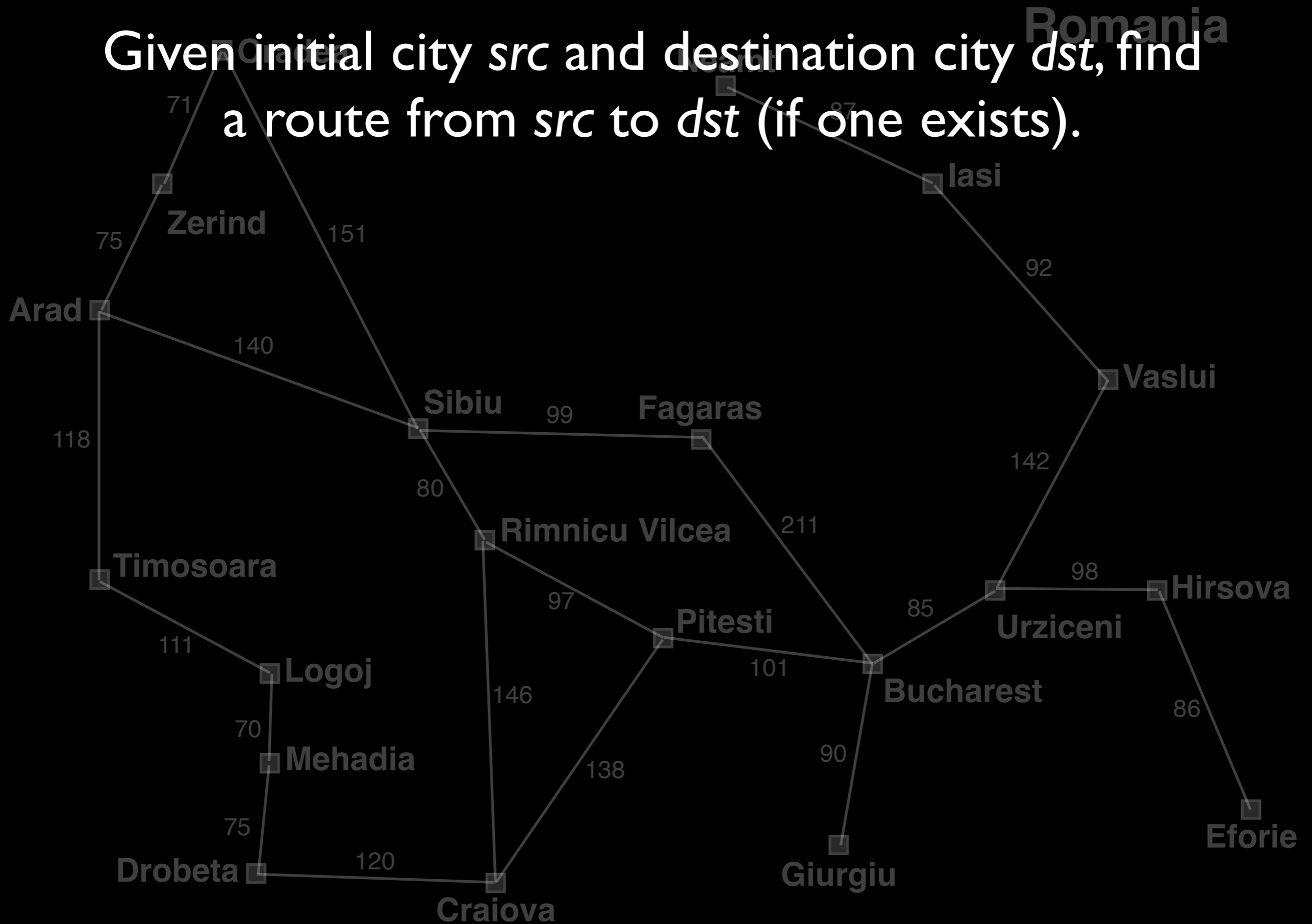
Romania



Romania



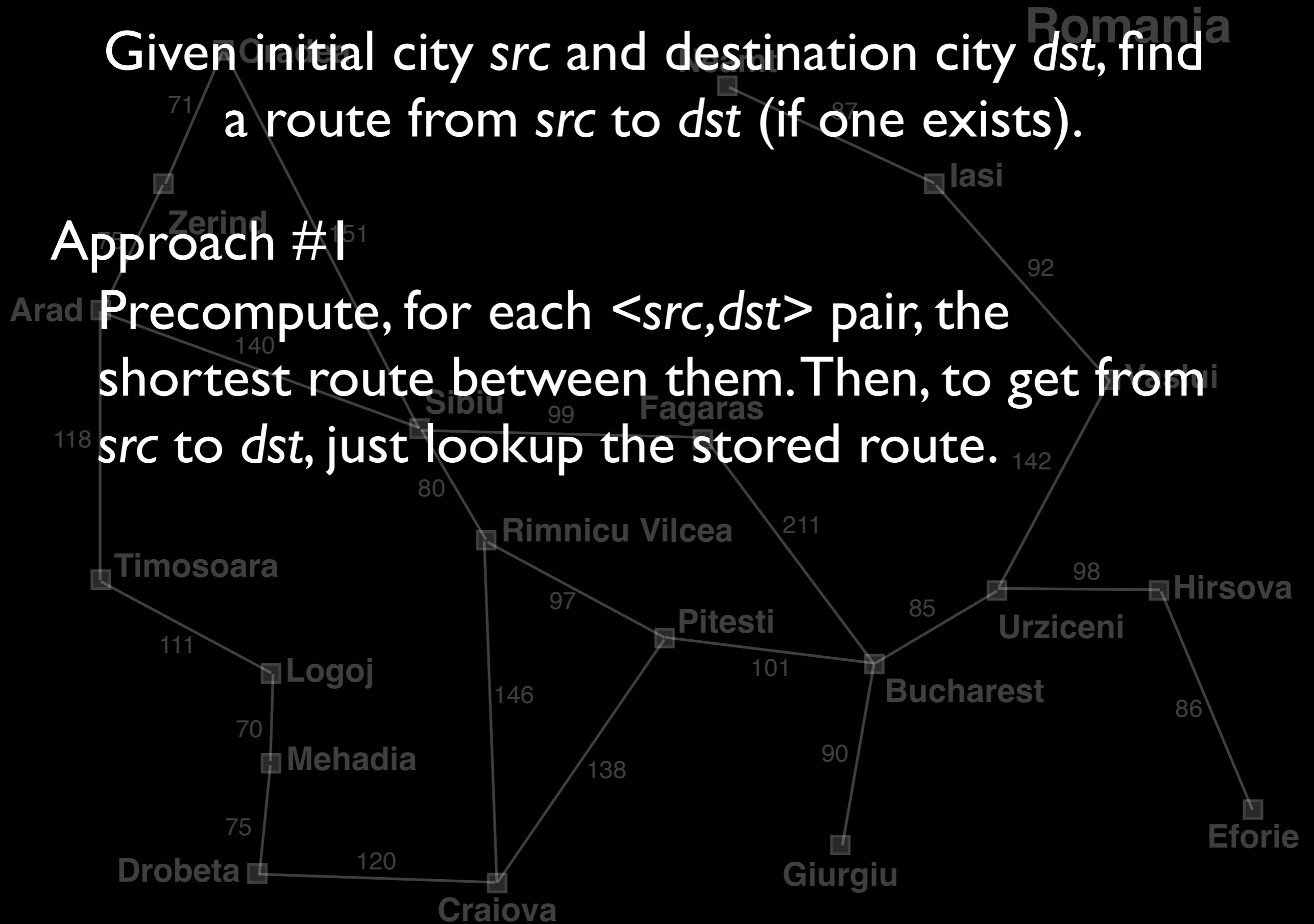
Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

Approach #1

Precompute, for each $\langle src, dst \rangle$ pair, the shortest route between them. Then, to get from *src* to *dst*, just lookup the stored route.



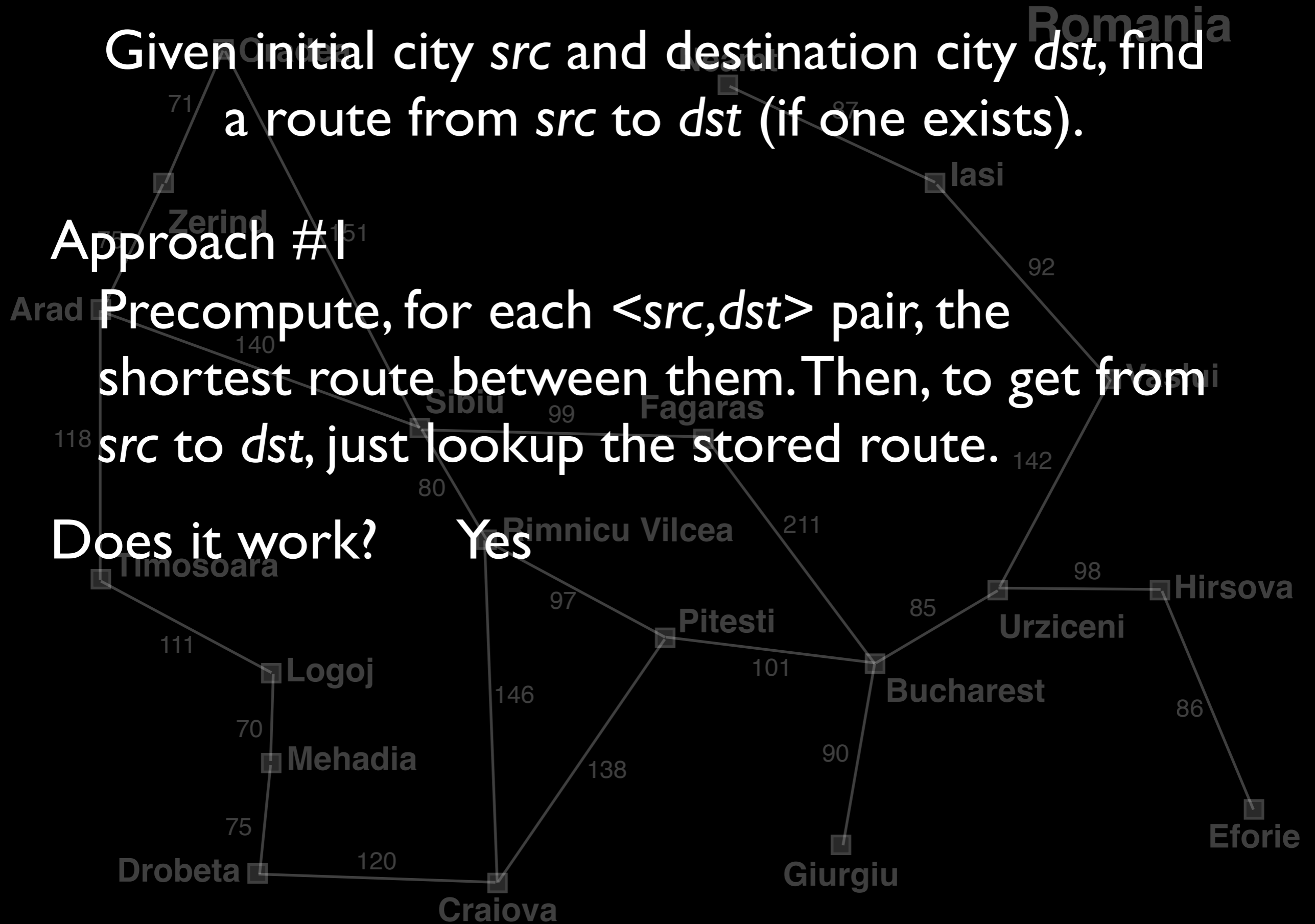
Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

Approach #1

Precompute, for each $\langle src, dst \rangle$ pair, the shortest route between them. Then, to get from *src* to *dst*, just lookup the stored route.

Does it work?

Yes



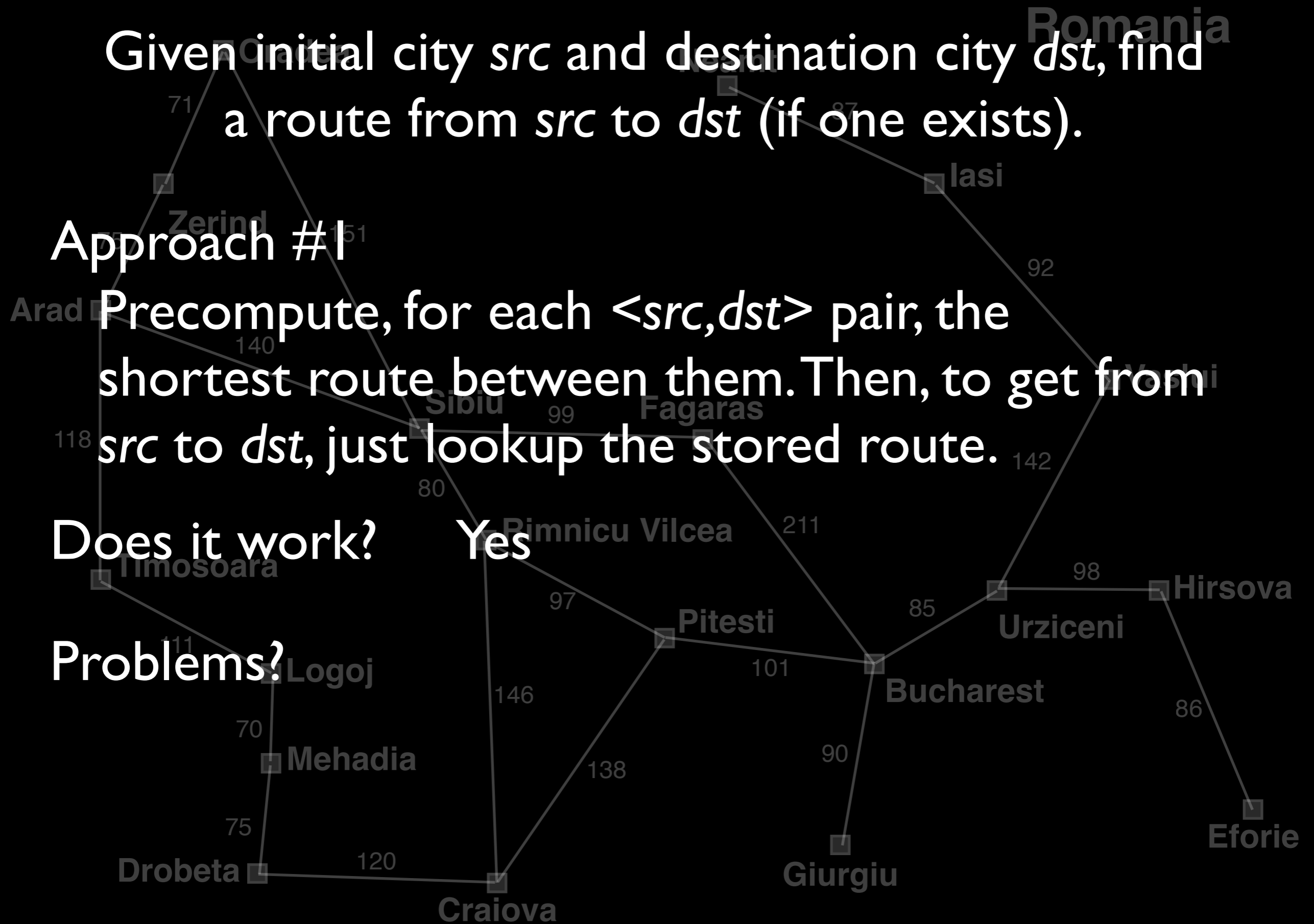
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Precompute, for each $\langle src, dst \rangle$ pair, the shortest route between them. Then, to get from *src* to *dst*, just lookup the stored route.

Does it work? Yes

Problems?



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

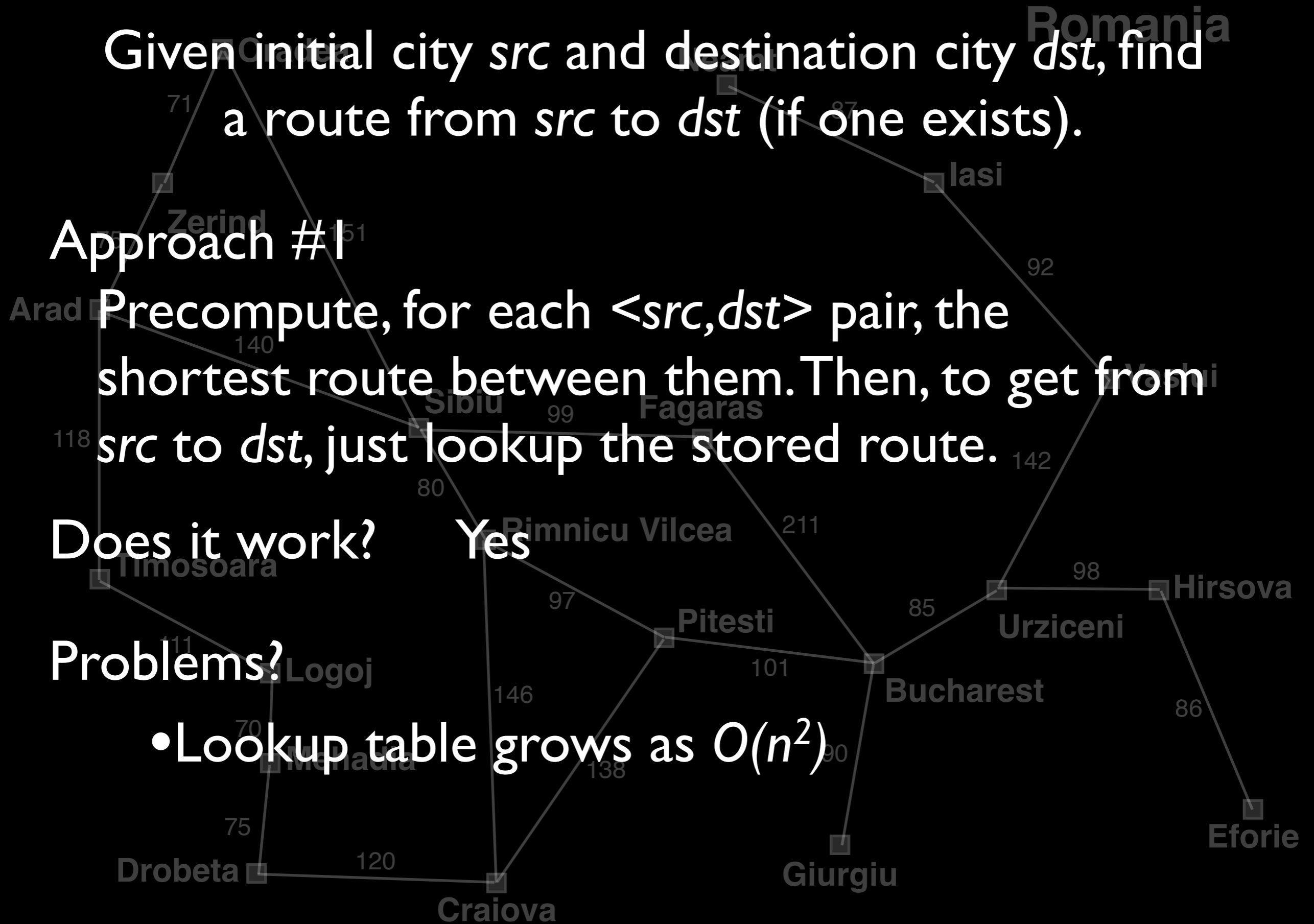
Approach #1

Precompute, for each $\langle src, dst \rangle$ pair, the shortest route between them. Then, to get from *src* to *dst*, just lookup the stored route.

Does it work? Yes

Problems?

- Lookup table grows as $O(n^2)$



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

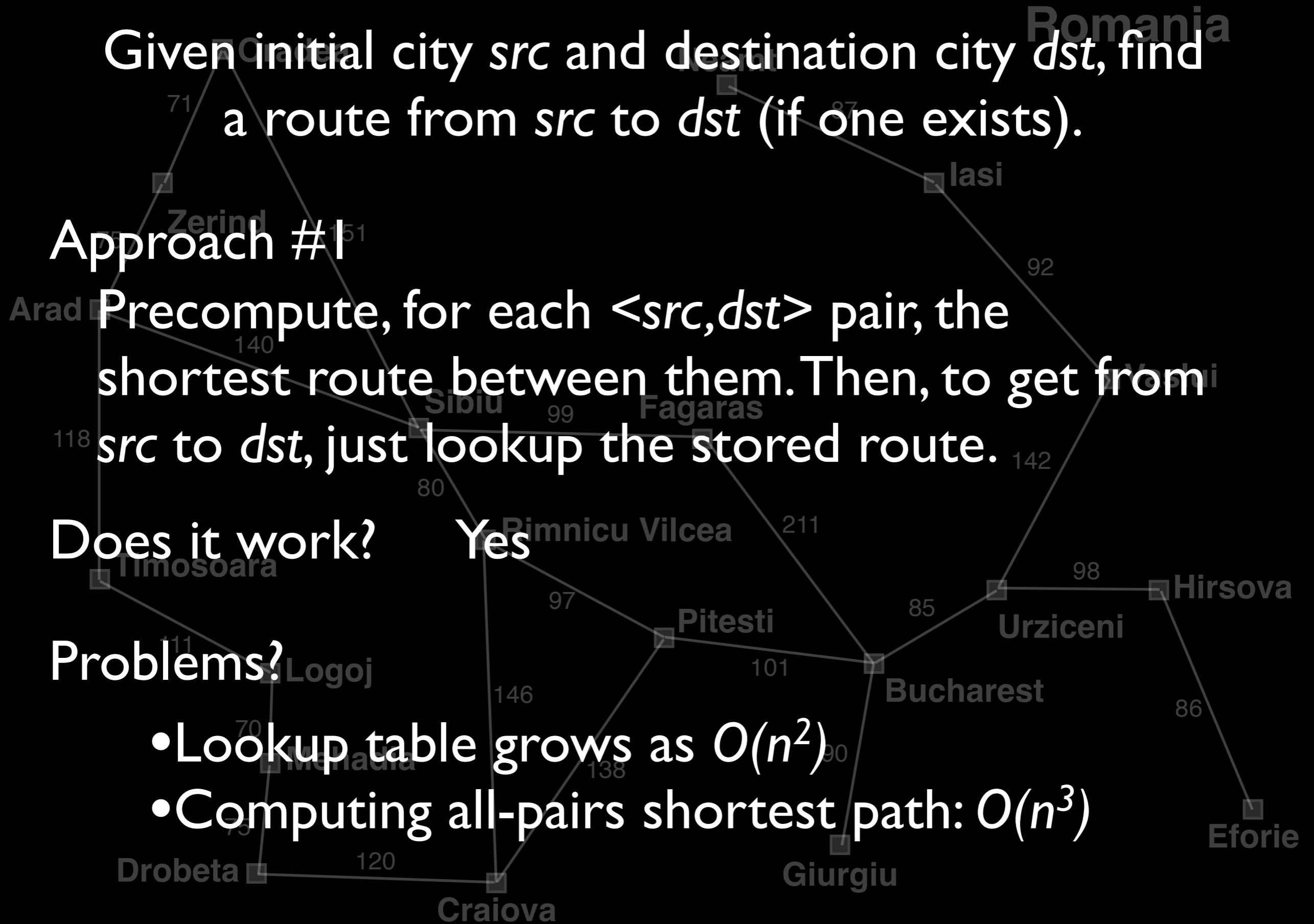
Approach #1

Precompute, for each $\langle src, dst \rangle$ pair, the shortest route between them. Then, to get from *src* to *dst*, just lookup the stored route.

Does it work? Yes

Problems?

- Lookup table grows as $O(n^2)$
- Computing all-pairs shortest path: $O(n^3)$



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

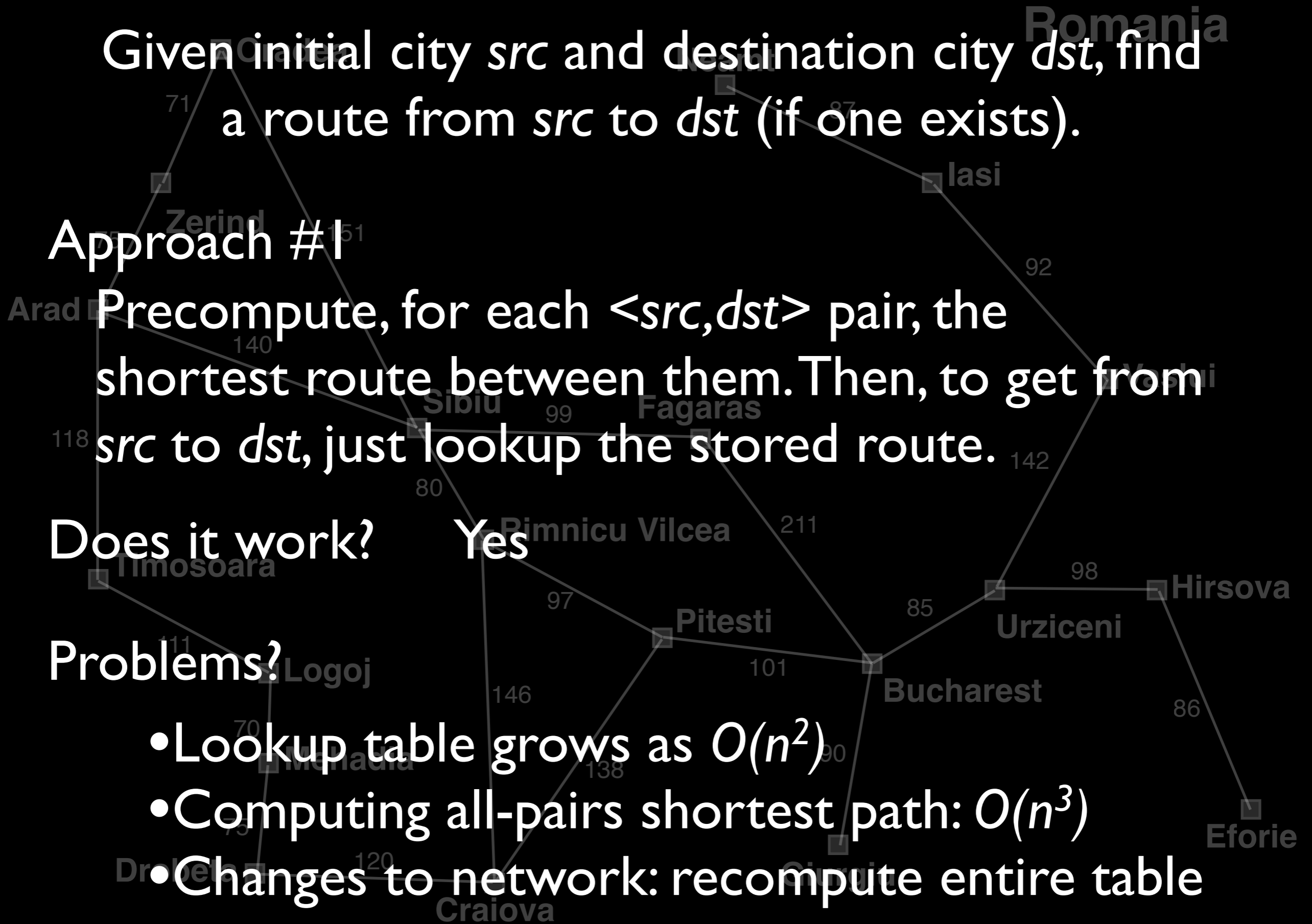
Approach #1

Precompute, for each $\langle src, dst \rangle$ pair, the shortest route between them. Then, to get from *src* to *dst*, just lookup the stored route.

Does it work? Yes

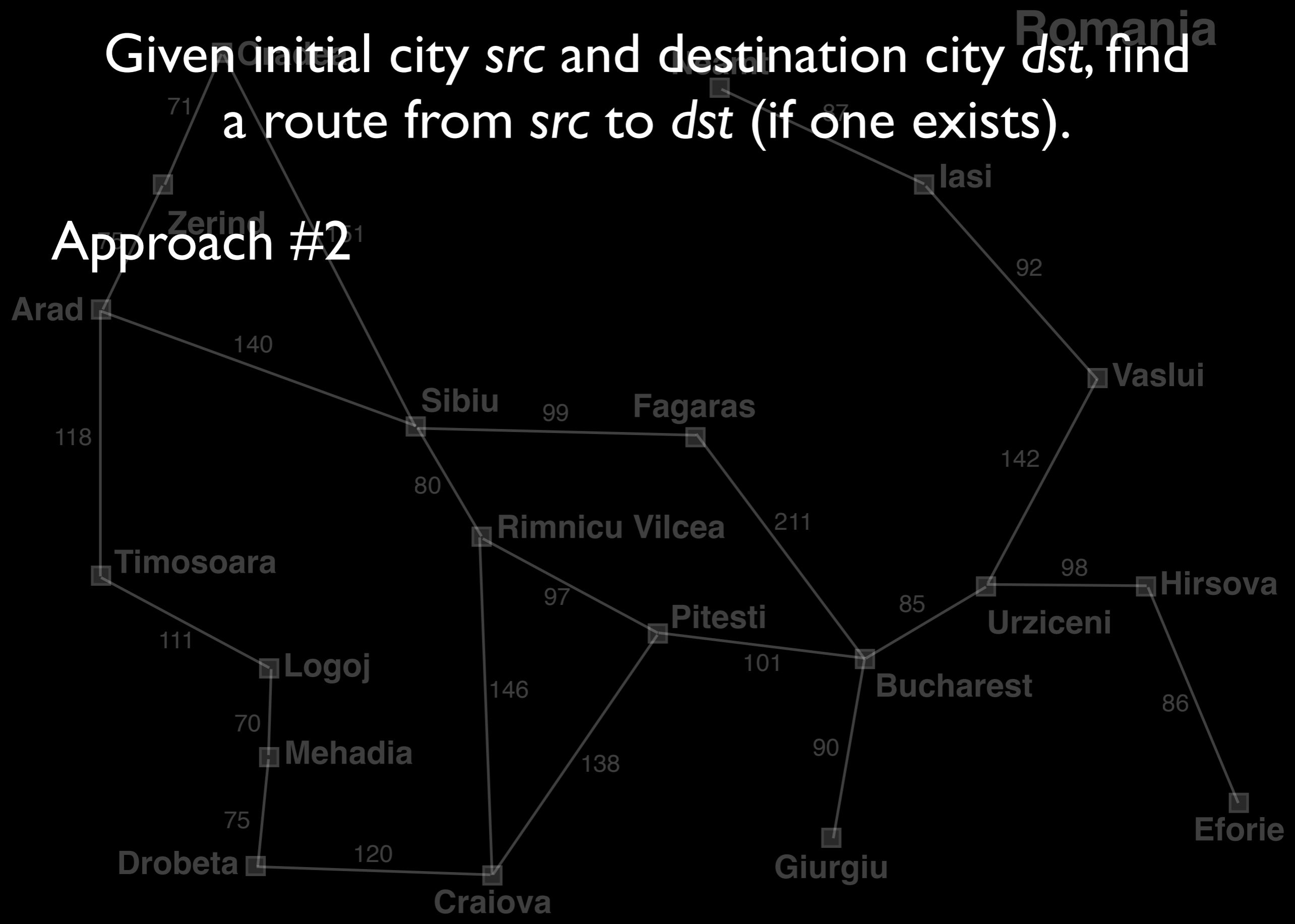
Problems?

- Lookup table grows as $O(n^2)$
- Computing all-pairs shortest path: $O(n^3)$
- Changes to network: recompute entire table



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

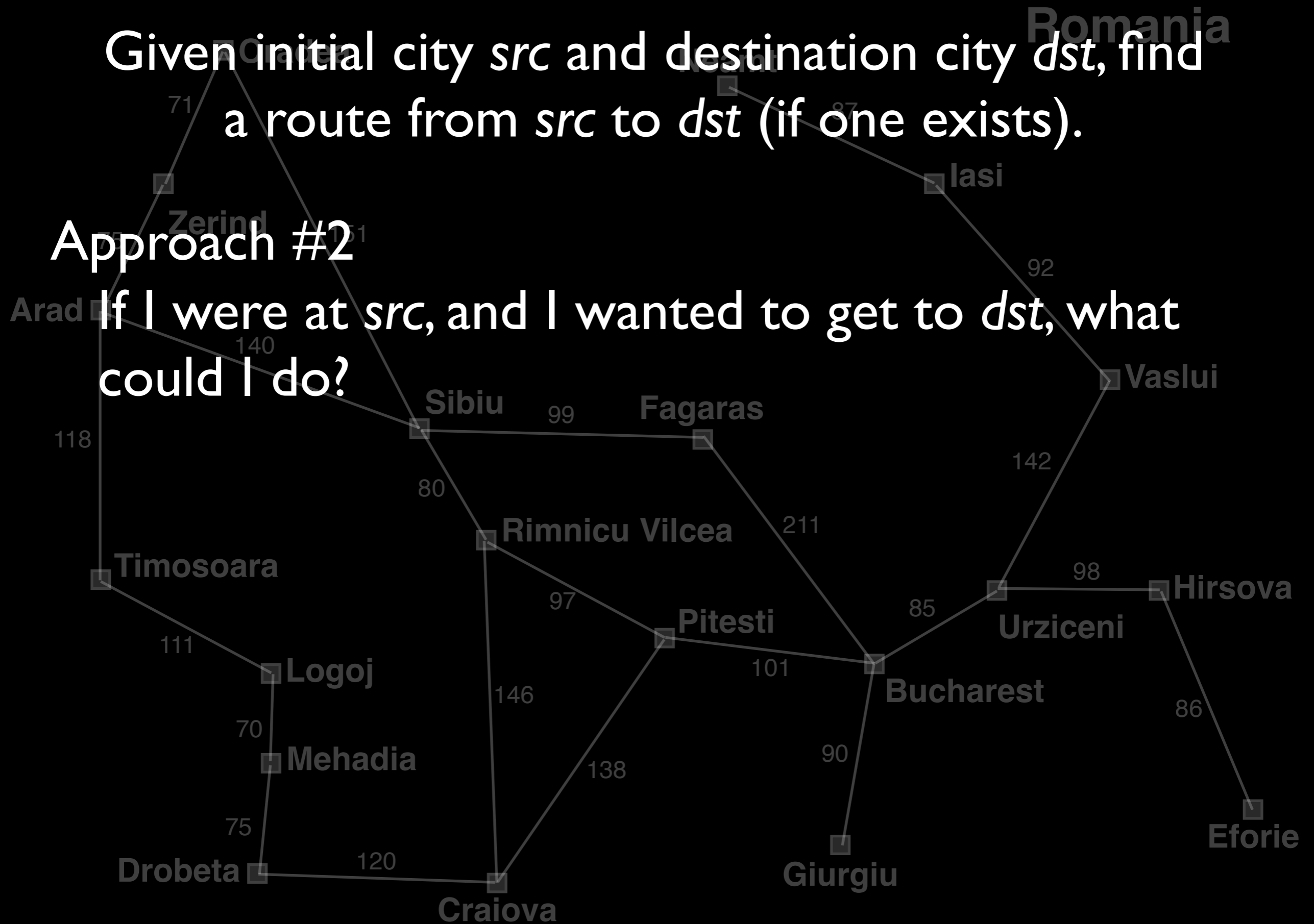
Approach #2



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

Approach #2

If I were at *src*, and I wanted to get to *dst*, what could I do?

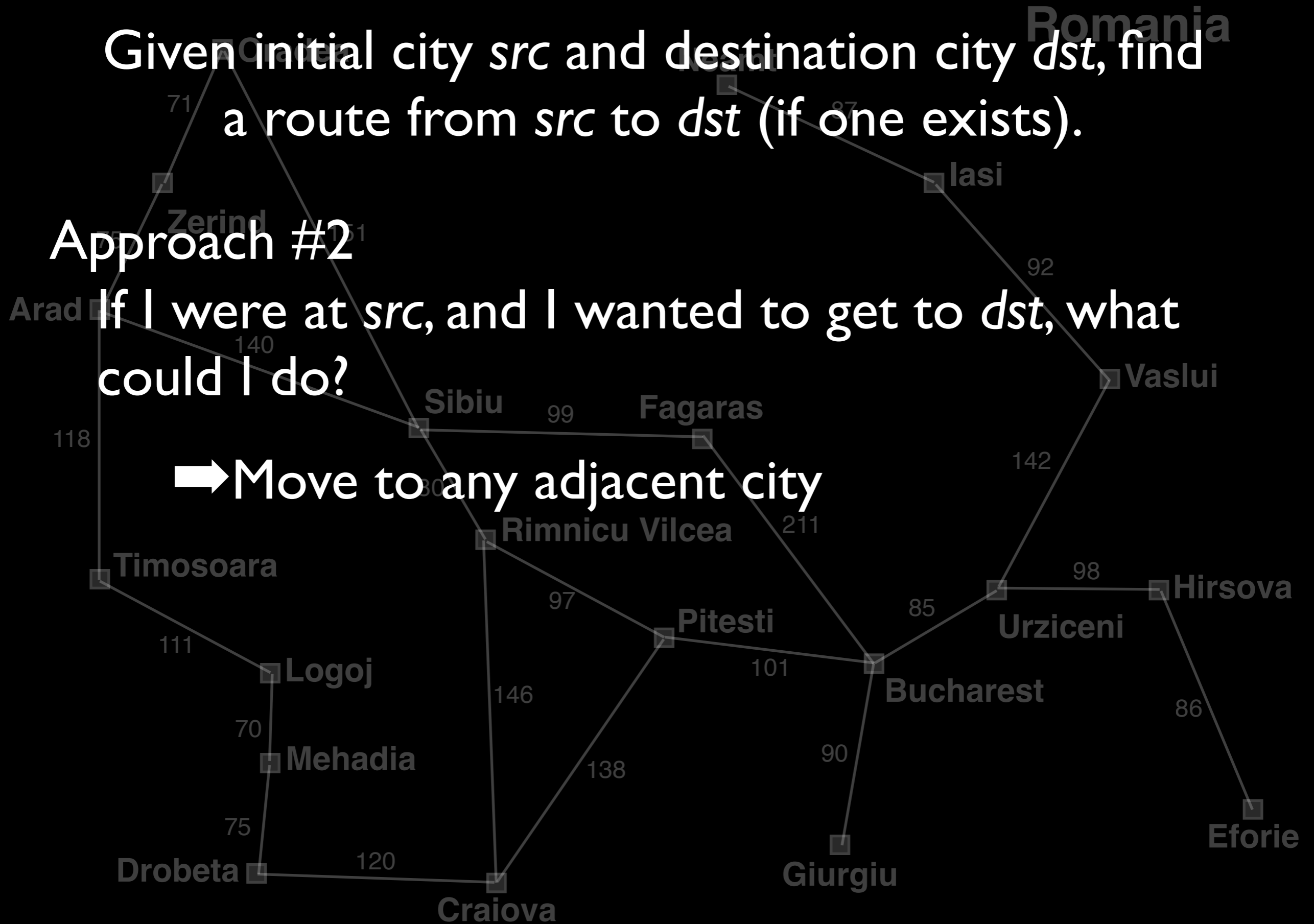


Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

Approach #2

If I were at *src*, and I wanted to get to *dst*, what could I do?

➡ Move to any adjacent city

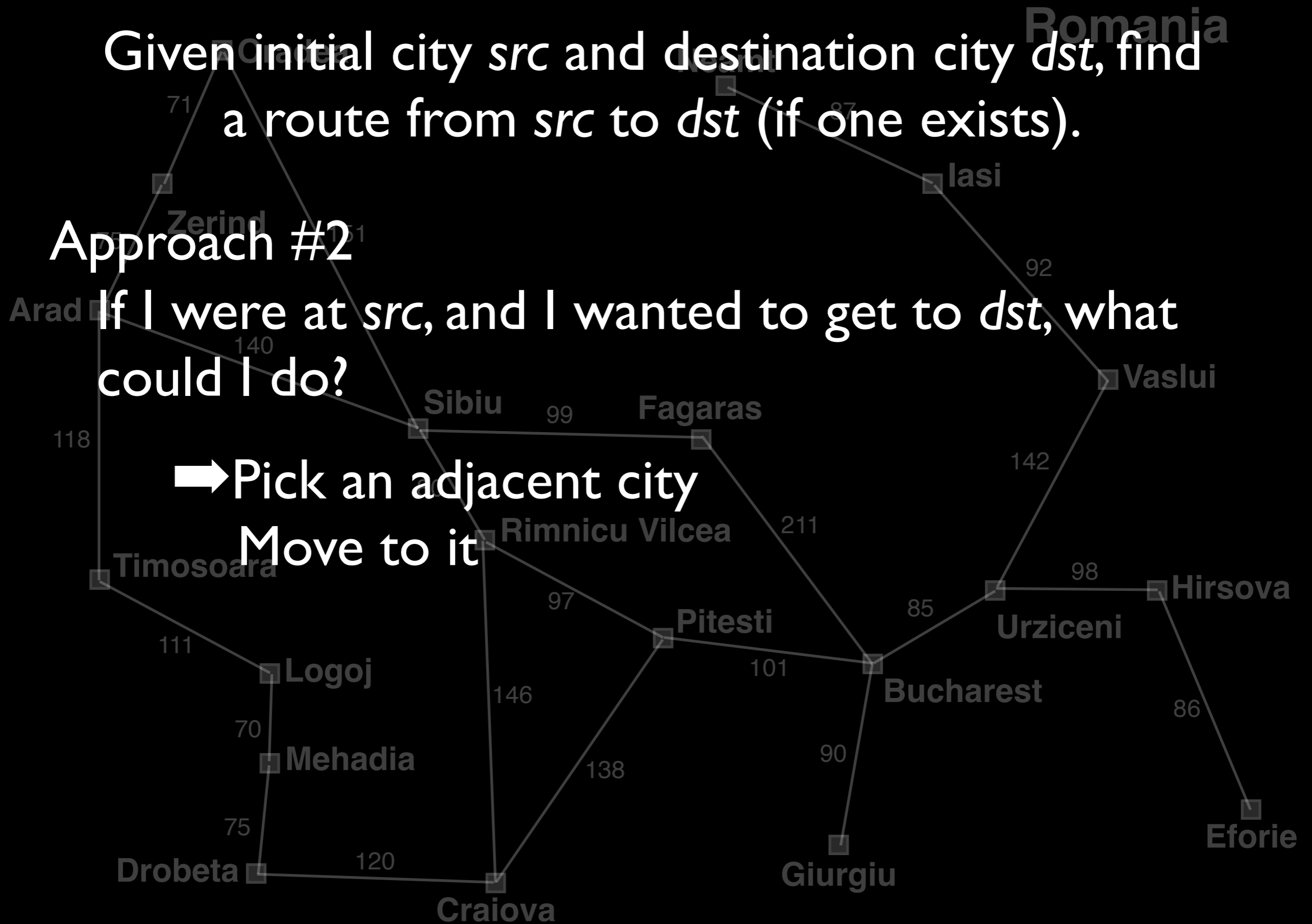


Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

Approach #2

If I were at *src*, and I wanted to get to *dst*, what could I do?

➔ Pick an adjacent city
Move to it



Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

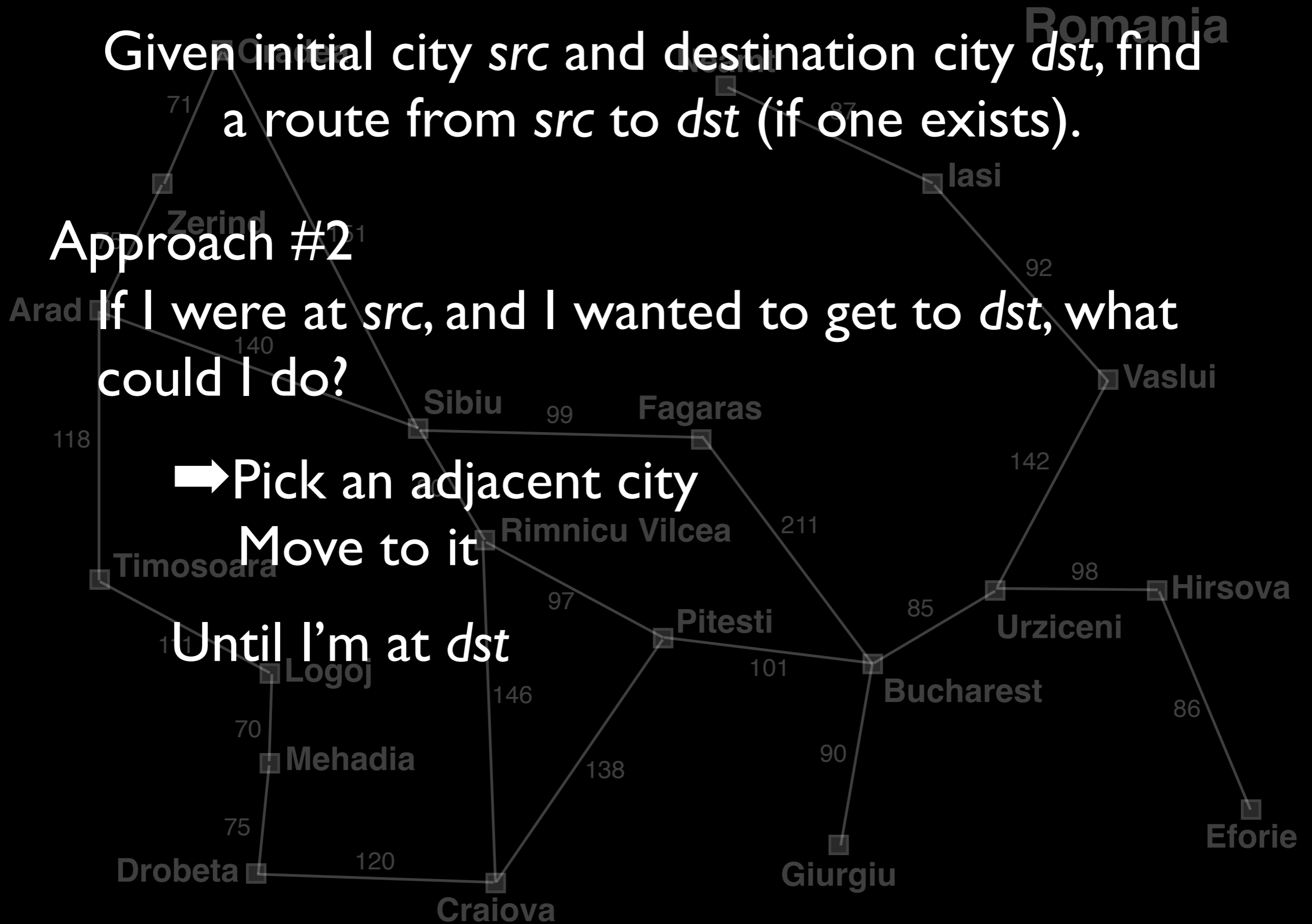
Approach #2

If I were at *src*, and I wanted to get to *dst*, what could I do?

➔ Pick an adjacent city

Move to it

Until I'm at *dst*



Romania



```
find_solution(City src, City dst) {
    City[] solution = [];
    City c = src;
    while (c != dst) {
        City[] neighbors = adjacent_cities(c);
        City next_c = select_one(neighbors);
        solution.add(next_c);
        c = next_c;
    }
    return solution;
}
```



```
find_solution(City src, City dst) {
    City[] solution = [];
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        c = next_c;
    }
    return solution;
}
```

Problems?

Intelligence and Generality

- Intelligence includes the ability to solve many kinds of problems
- Including problems we haven't seen before
- Every new problem-solving method needs to be designed, implemented, tested, and debugged



1	2	3
4	5	6
7	8	

Given initial puzzle configuration *start* and desired configuration *goal*, find a sequence of moves that goes from *start* to *goal* (if one exists).

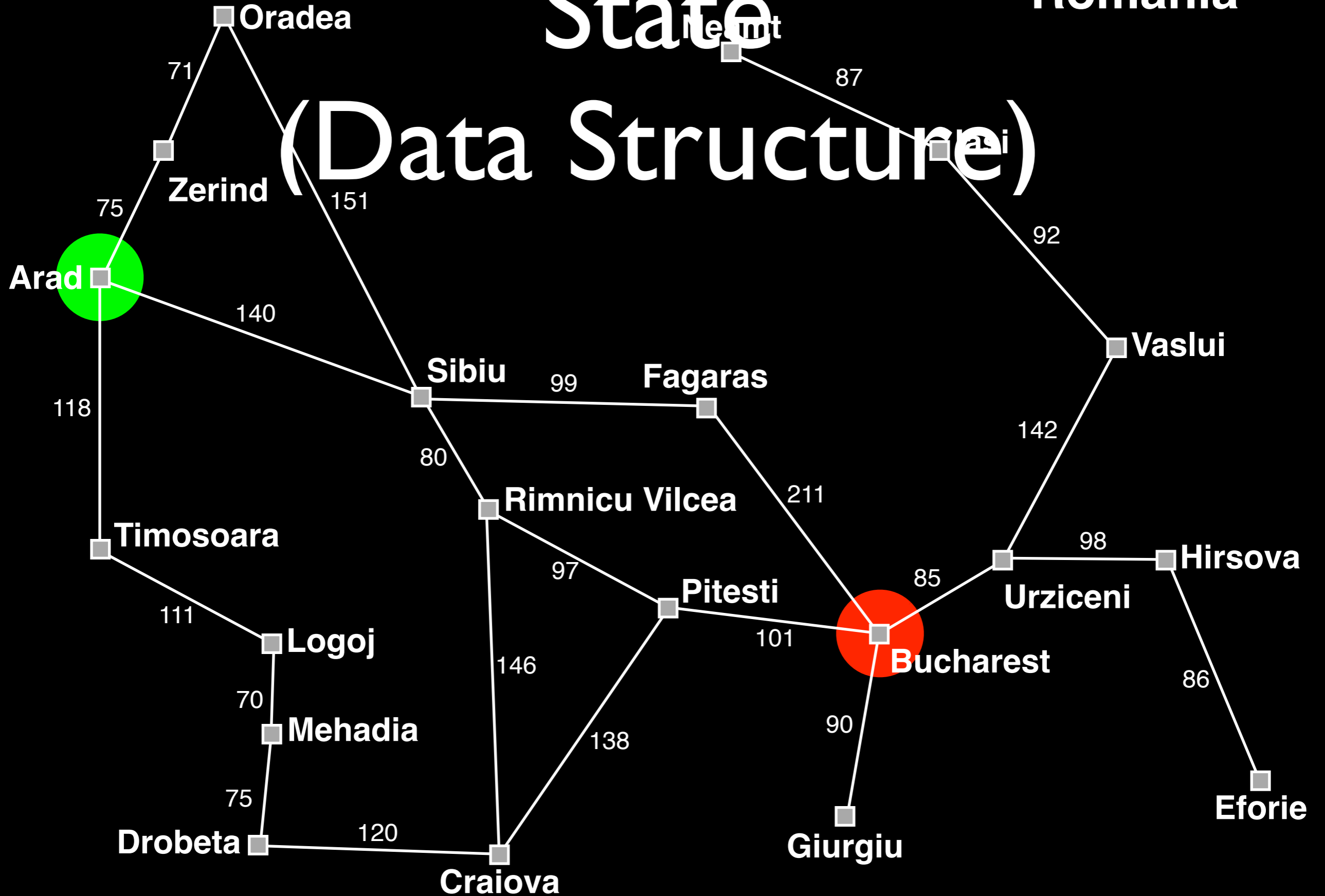
1	2	3
4	5	6
7	8	

State (Data Structure)

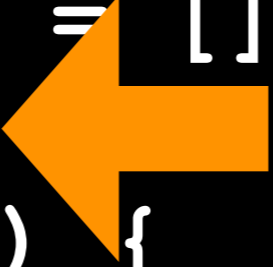
State

Romania

(Data Structure)



```
find_solution(City src, City dst) {
    City[] solution = [];
    City c = src;
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    }
    return solution;
}
```



State (Data Structure)

7		1
6	2	8
3	4	5

State (Data Structure)

7		1
6	2	8
3	4	5

$$M_{i,j} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix}$$

State (Data Structure)

7		1
6	2	8
3	4	5

$$M_{i,j} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix}$$


```
int[][] M = new int[3][3];  
M[0][0] = 7;  
M[0][1] = 0;  
...  
M[2][2] = 5;
```

Actions

- Can be performed in a state
- Change the state to a resulting state

Romania



```
find_solution(City src, City dst) {  
    City[] solution = [];  
    City c = src;  
    while (c != dst) {  
        City[] neighbors = adjacent_cities(c);  
        City next_c = select_one(neighbors);  
        solution.add(next_c);  
        c = next_c;   
    }  
    return solution;  
}
```

7		1
6	2	8
3	4	5

Path:

7	1	
6	2	8
3	4	5

Path: I

7	1	8
6	2	
3	4	5

Path: 1 - 8

7	1	8
6		2
3	4	5

Path: 1 - 8 - 2

7	1	8
	6	2
3	4	5

Path: 1 - 8 - 2 - 6

7		1
6	2	8
3	4	5

Path:

7	1	
6	2	8
3	4	5

Path: East

7	1	8
6	2	
3	4	5

Path: East - South

7	1	8
6		2
3	4	5

Path: East - South - West -

7	1	8
	6	2
3	4	5

Path: East - South - West - West

Actions

- For any state and action:
 - Can I perform this action in this state?
 - “Applicability”
 - How do I update the state if this action is performed?
 - “Result” or “transition” function

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

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Approach #1

Precompute, for each $\langle M^{\text{in}}, M^{\text{G}} \rangle$ pair, a sequence of moves that transforms M^{in} into M^{G} . Then, to solve M^{in} , just lookup the stored sequence.

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #1

Precompute, for each $\langle M^{\text{in}}, M^{\text{G}} \rangle$ pair, a sequence of moves that transforms M^{in} into M^{G} . Then, to solve M^{in} , just lookup the stored sequence.

Does it work?

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #1

Precompute, for each $\langle M^{\text{in}}, M^{\text{G}} \rangle$ pair, a sequence of moves that transforms M^{in} into M^{G} . Then, to solve M^{in} , just lookup the stored sequence.

Does it work? No

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

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Does it work? No

Problems?

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

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Precompute, for each $\langle M^{\text{in}}, M^{\text{G}} \rangle$ pair, a sequence of moves that transforms M^{in} into M^{G} . Then, to solve M^{in} , just lookup the stored sequence.

Does it work? No

Problems?

- There are $9! = 362880$ ($O(n!)$ in general) cases

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #1

Precompute, for each $\langle M^{\text{in}}, M^{\text{G}} \rangle$ pair, a sequence of moves that transforms M^{in} into M^{G} . Then, to solve M^{in} , just lookup the stored sequence.

Does it work? No

Problems?

- There are $9! = 362880$ ($O(n!)$ in general) cases
- No obvious way to solve the $\langle M^{\text{in}}, M^{\text{G}} \rangle$ cases

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #1

Precompute, for each $\langle M^{\text{in}}, M^{\text{G}} \rangle$ pair, a sequence of moves that transforms M^{in} into M^{G} . Then, to solve M^{in} , just lookup the stored sequence.

Does it work? No

Problems?

- There are $9! = 362880$ ($O(n!)$ in general) cases
- No obvious way to solve the $\langle M^{\text{in}}, M^{\text{G}} \rangle$ cases
- This problem is known to be NP-complete

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #2

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #2

If the puzzle is currently M , and I want it to be M^{G} , what could I do?

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #2

If the puzzle is currently M , and I want it to be M^{G} , what could I do?

➡ Move the blank to an adjacent space

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #2

If the puzzle is currently M , and I want it to be M^{G} , what could I do?

- ➔ Pick an adjacent space
- Move the blank to it

$$M^{\text{in}} = \begin{bmatrix} 7 & 0 & 1 \\ 6 & 2 & 8 \\ 3 & 4 & 5 \end{bmatrix} \quad M^{\text{G}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

Approach #2

If the puzzle is currently M , and I want it to be M^{G} , what could I do?

➡ Pick an adjacent space
Move the blank to it

Until $M == M^{\text{G}}$

Given initial city *src* and destination city *dst*, find a route from *src* to *dst* (if one exists).

Approach #2

If I were at *src*, and I wanted to get to *dst*, what could I do?

➡ Pick an adjacent city
Move to it

Until I'm at *dst*

Initial state: S^{IN}

Goal state: S^G

If the problem state is currently S , and I want it to be S^G , what could I do?

➔ Pick an applicable action A

Update S with the result of applying A

Until $S == S^G$

```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
    while (!is_goal(s)) {  
        Action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

state

```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
    while (!is_goal(s)) {  
        Action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

initial state

```
find_solution(State initial) {  
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    Action[] solution = [];  
    while (!is_goal(s)) {  
        Action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

```
find_solution(State initial, State goal) {
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    Action[] solution = [];
    while (!is_goal(s)) {
        Action a = pick(actions(s));
        solution.add(a);
        s = result(s, a);
    }
    return solution;
}
```

goal test

```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
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        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```



action


```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
    while (!is_goal(s)) {  
        Action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```



applicable actions

```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
    while (!is_goal(s)) {  
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        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```



transition function

```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
    while (!is_goal(s)) {  
        Action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```



solution

```
find_solution(State initial) {  
    State s = initial;  
    Action[] solution = [];  
    while (!is_goal(s)) {  
        Action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

Problem (Domain): $\langle \mathcal{S}, \mathcal{A}, \text{ACTIONS}, \text{RESULT} \rangle$

$\text{ACTIONS} : s \in \mathcal{S} \rightarrow$

$\{a \in \mathcal{A} : a \text{ can be executed (is applicable) in } s\}$

$\text{RESULT} : s \in \mathcal{S}, a \in \mathcal{A} \rightarrow$

$s' : s' \in \mathcal{S} \text{ s.t. } s' \text{ is the result of performing } a \text{ in } s$

Problem (Instance): $\langle \mathcal{I} \in \mathcal{S}, \mathcal{G} \subseteq \mathcal{S} \rangle$

Solution: $\langle a_0, a_1, \dots, a_n \rangle \in \mathcal{A} \text{ s.t.}$

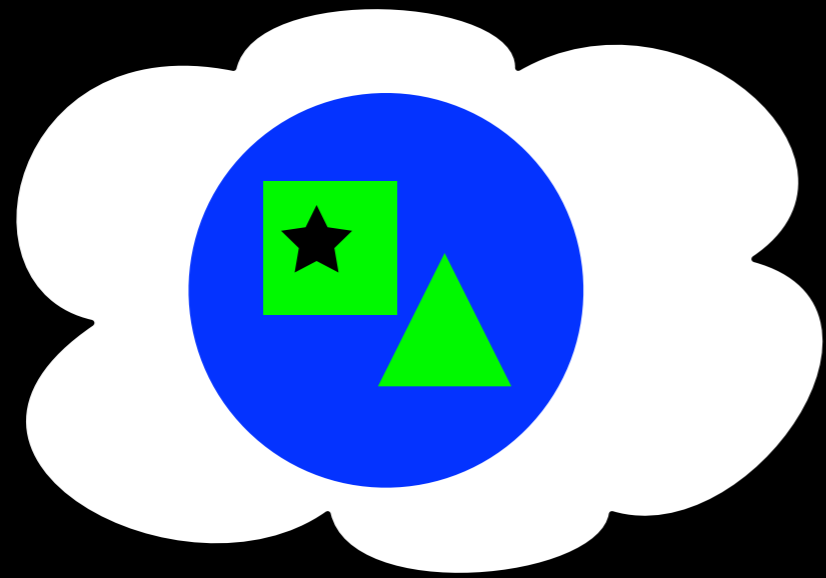
$\text{RESULT}(\dots \text{RESULT}(\text{RESULT}(\mathcal{I}, a_0), a_1) \dots, a_n) \in \mathcal{G}$



World state



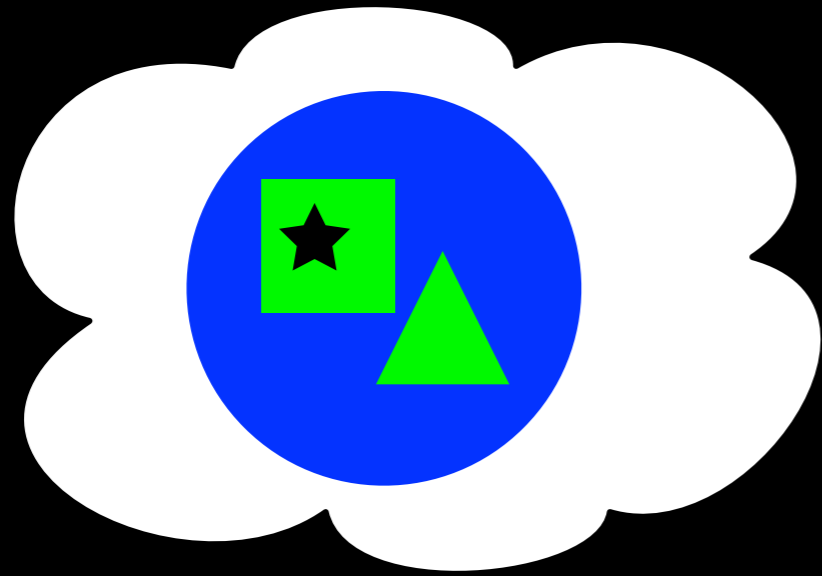
World state



Problem-solving state



World state

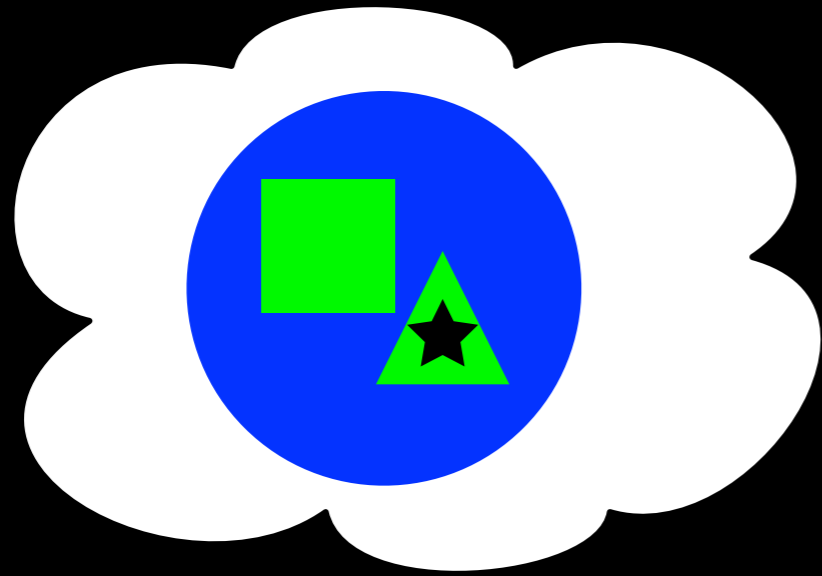


Problem-solving state



World state

Action



Problem-solving state
Transition Model



World state
Action

Universal Problem-Solving Procedure

```
find_solution(initial) {  
    state s = initial;  
    action[] solution = [];  
    while (!is_goal(s)) {  
        action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

Are We Done Yet?

```
find_solution(initial) {  
    state s = initial;  
    action[] solution = [];  
    while (!is_goal(s)) {  
        action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```




FAILARMY



FAILARMY

Are We Done Yet?

```
find_solution(initial) {  
    state s = initial;  
    action[] solution = [];  
    while (!is_goal(s)) {  
        action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

Are We Done Yet?

```
find_solution(initial) {  
    state s = initial;  
    action[] solution = [];  
    while (!is_goal(s)) {  
        action a = pick(actions(s));  
        solution.add(a);  
        s = result(s, a);  
    }  
    return solution;  
}
```

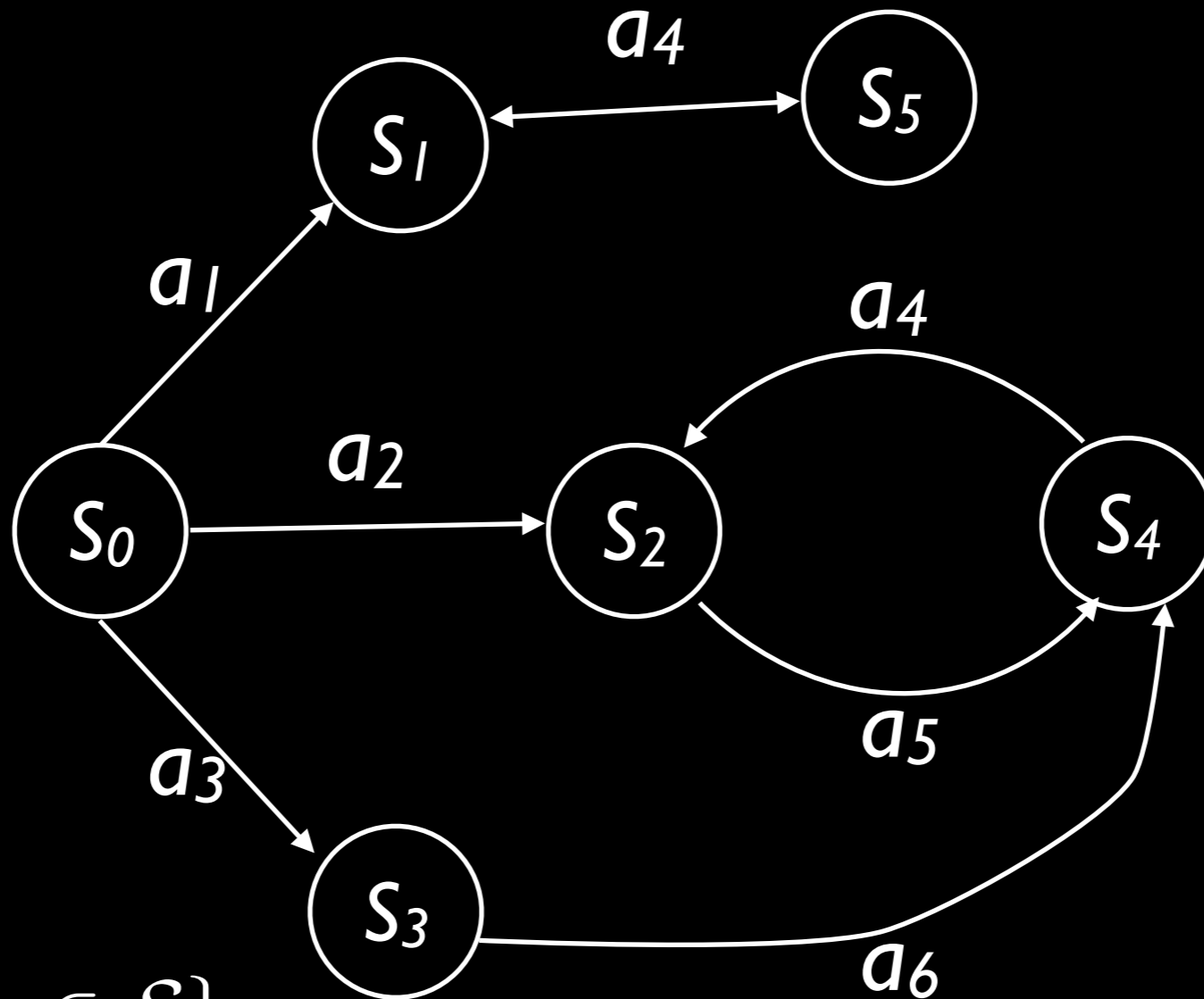
FAIL

State-Space Search

States + Actions + Transition Model
=
State Space

The set of all states reachable from the initial state by some sequence of actions

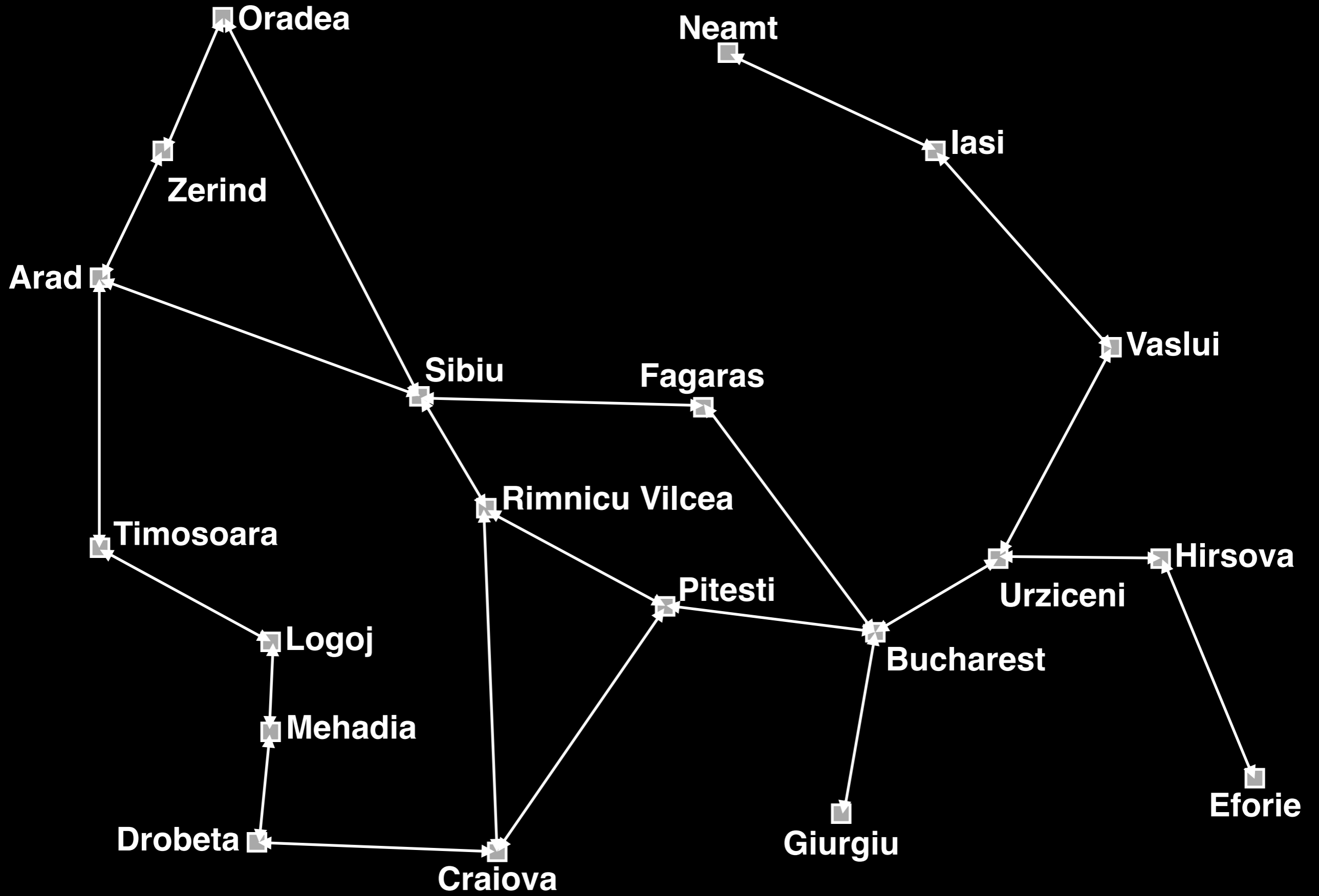
State Space



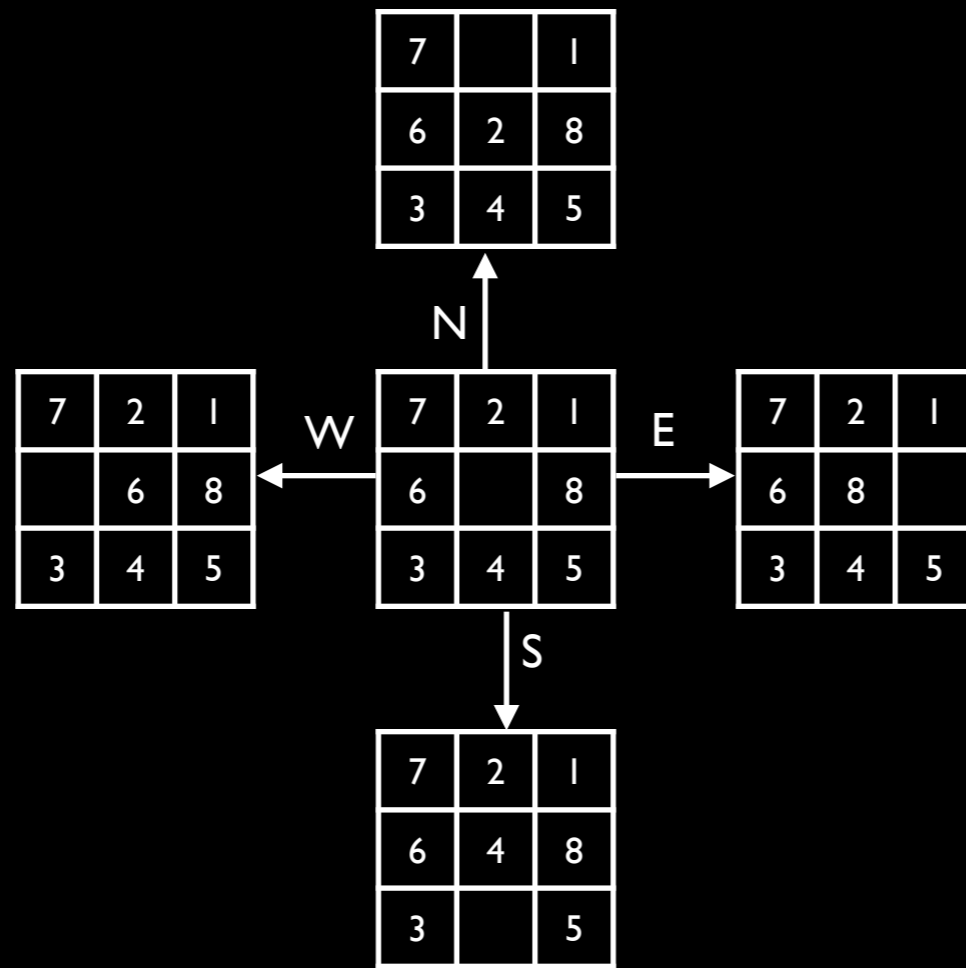
$\langle V, E \rangle :$

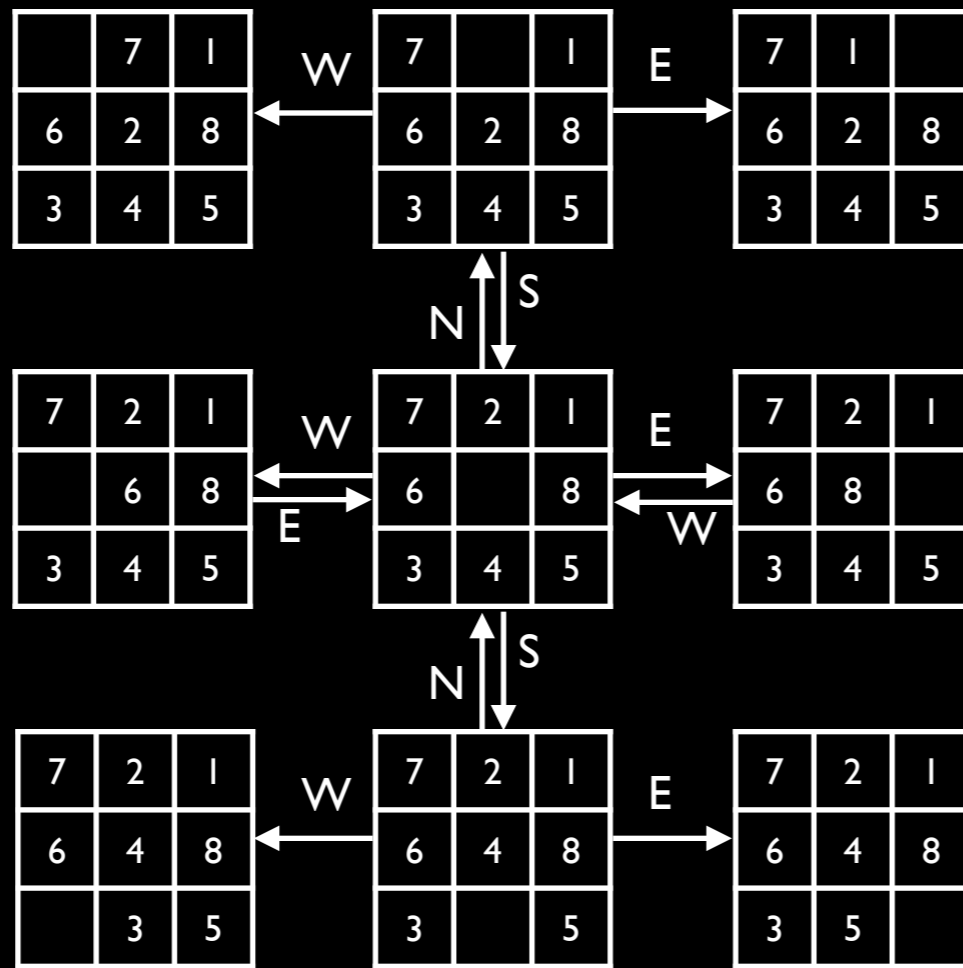
$$V = \{v_i \mid s_i \in \mathcal{S}\}$$

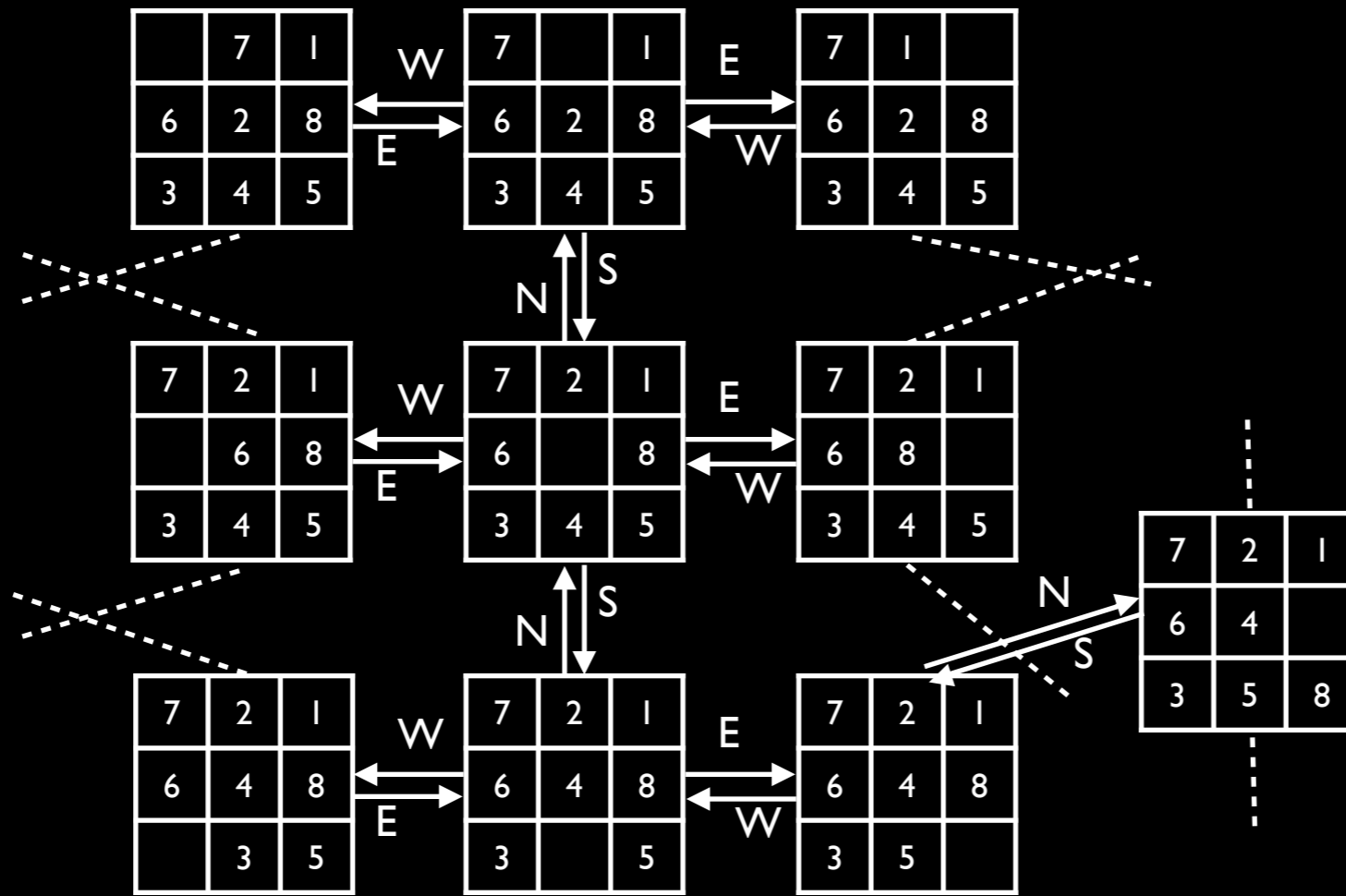
$$E = \{\langle v_i, v_j, a \rangle \mid s_j = \text{RESULT}(s_i, a)\}$$

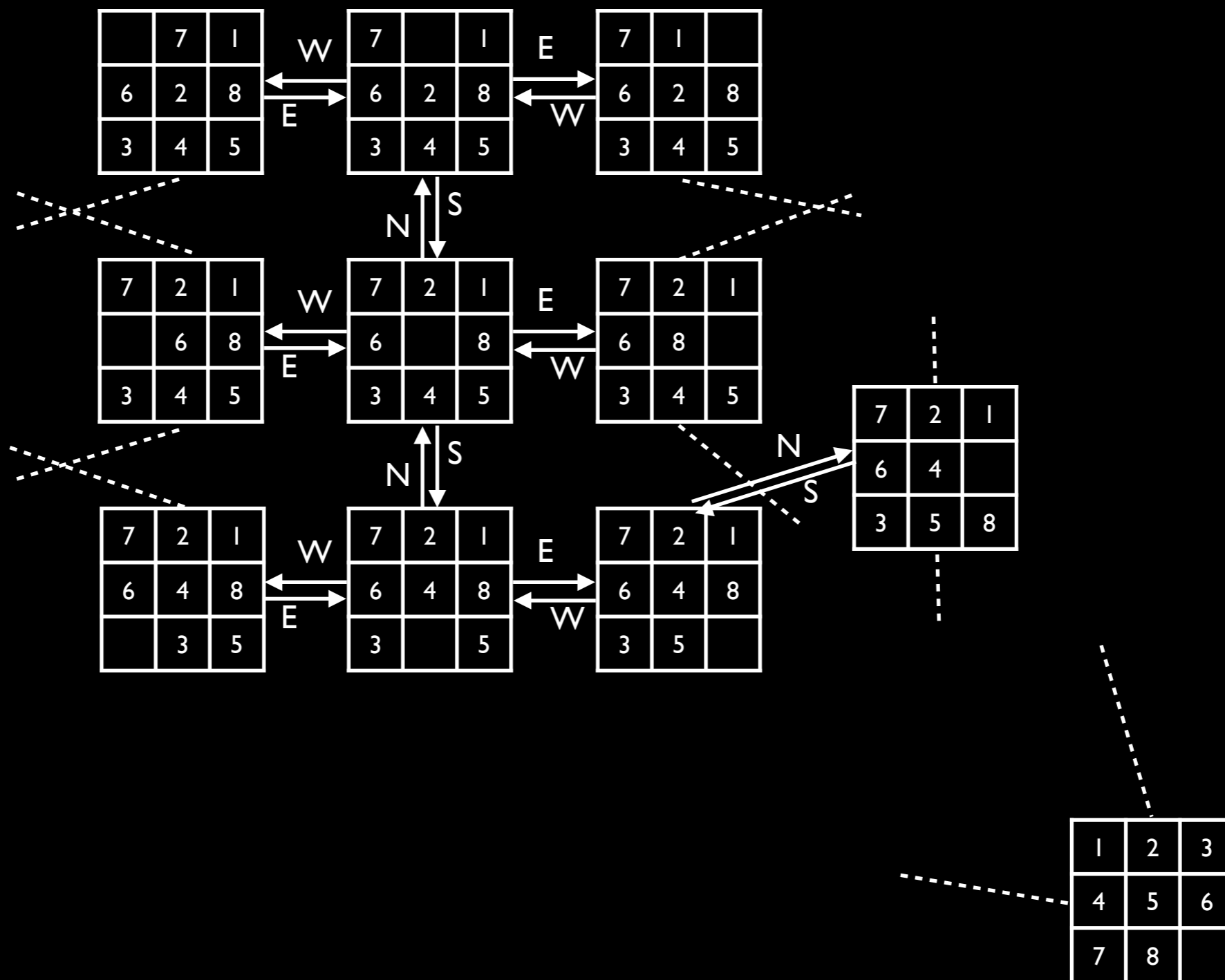


7	2	1
6		8
3	4	5

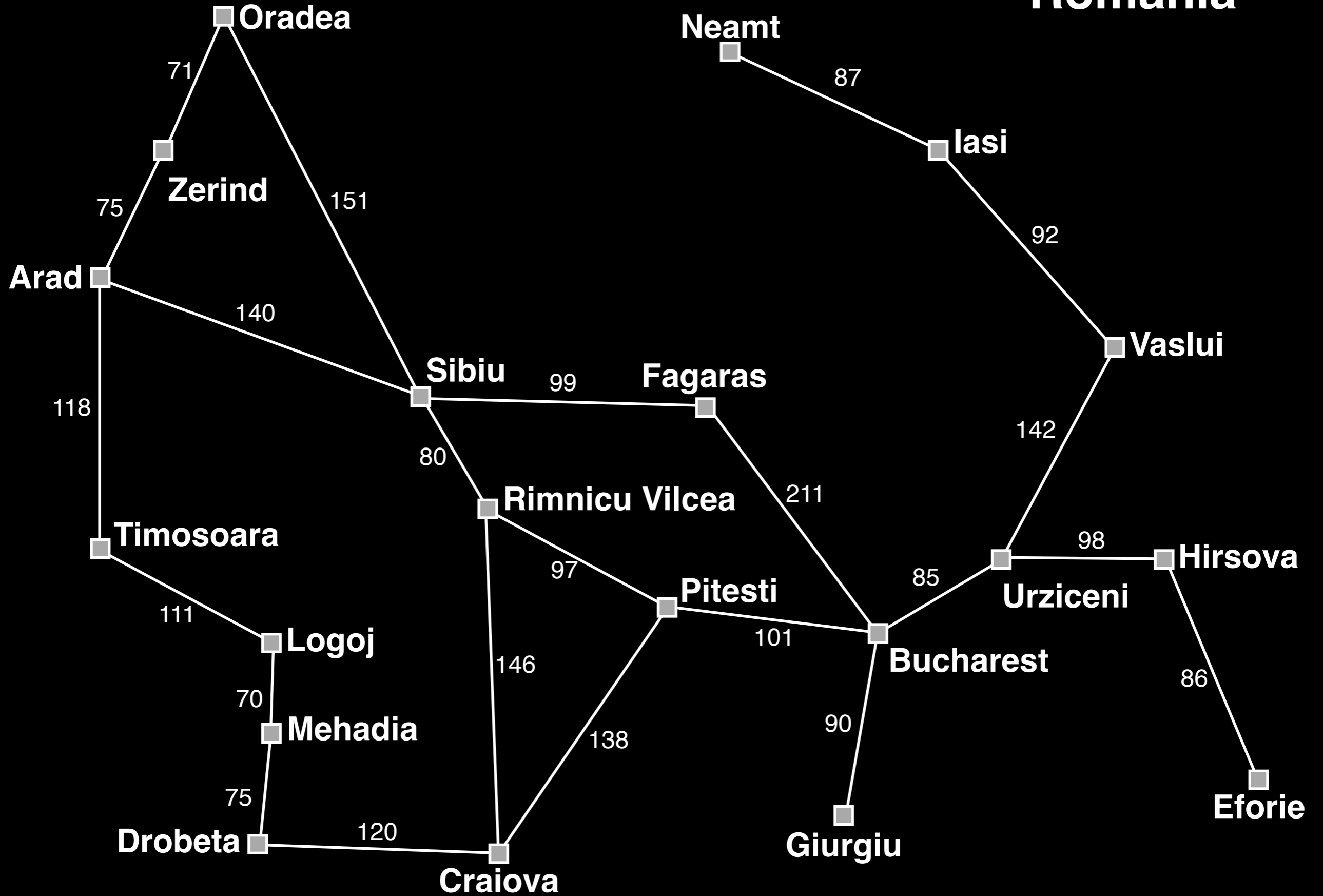




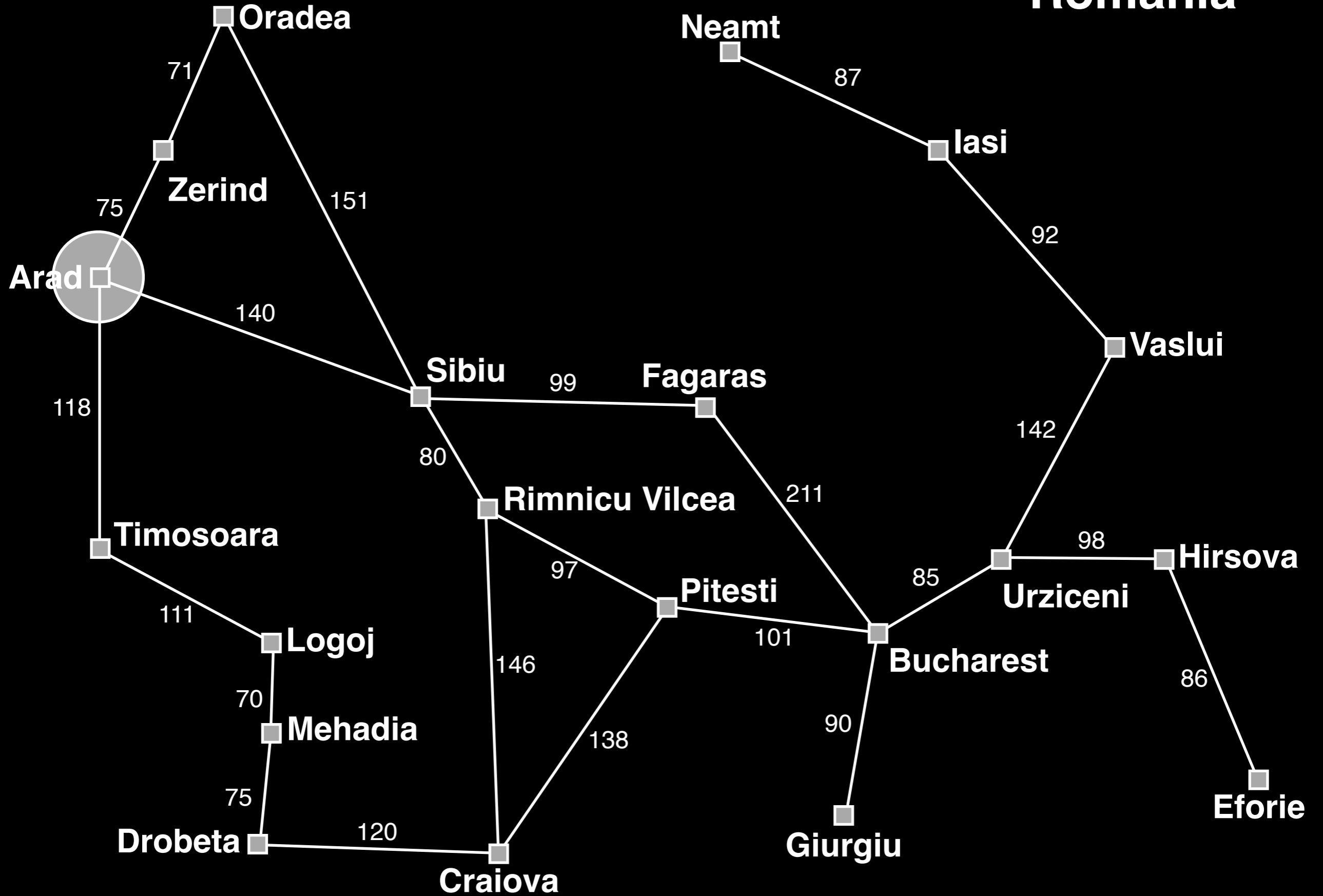




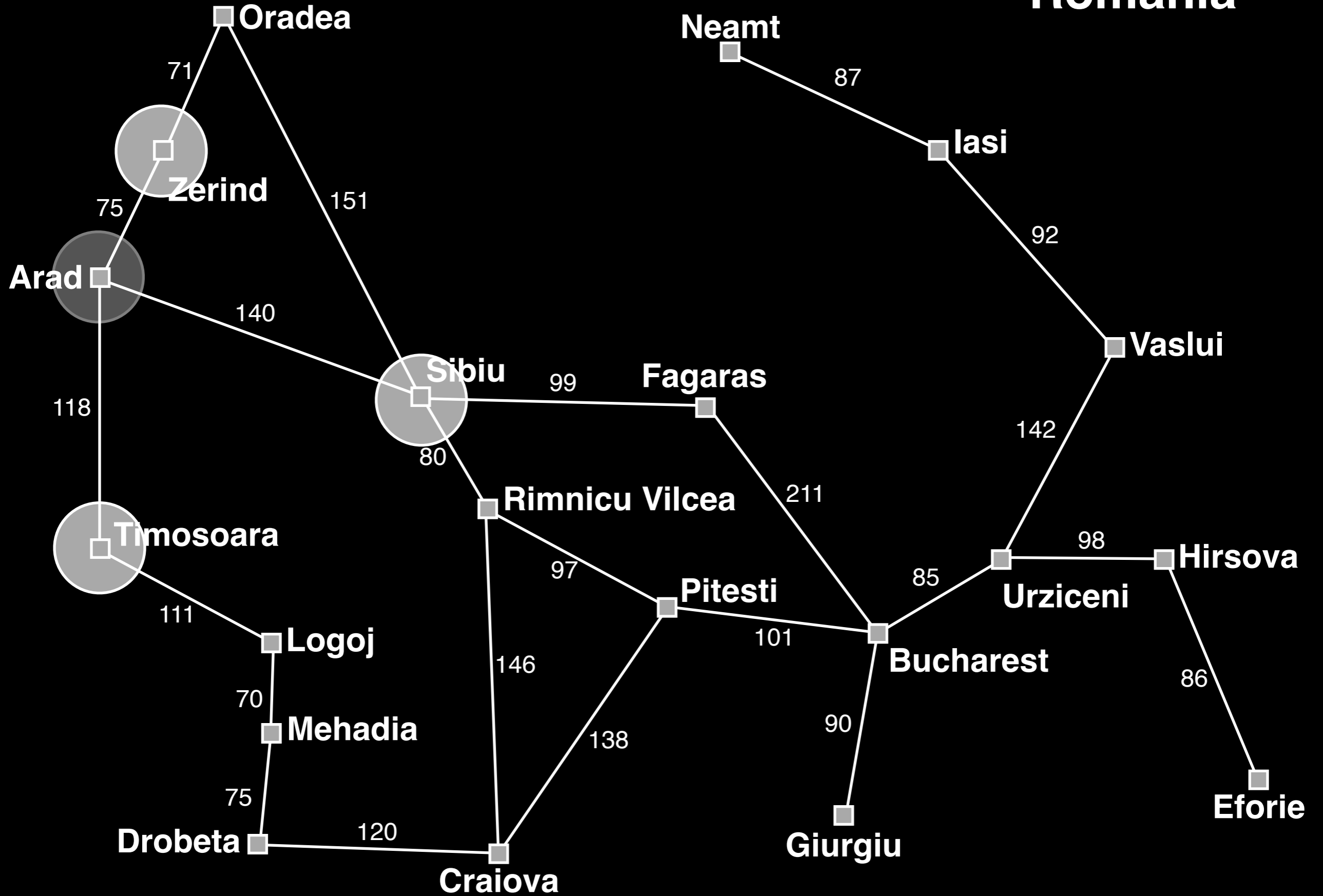
Romania



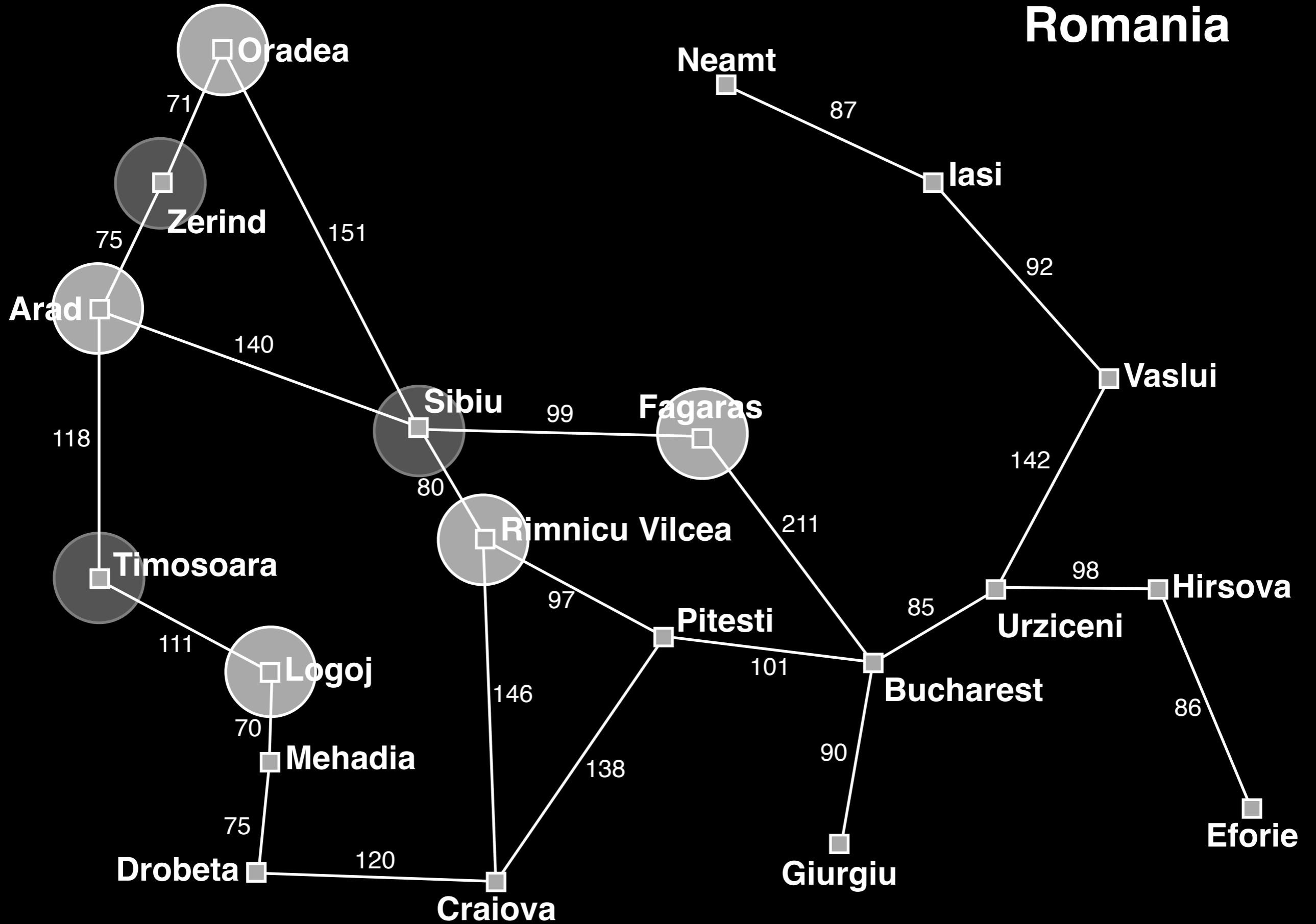
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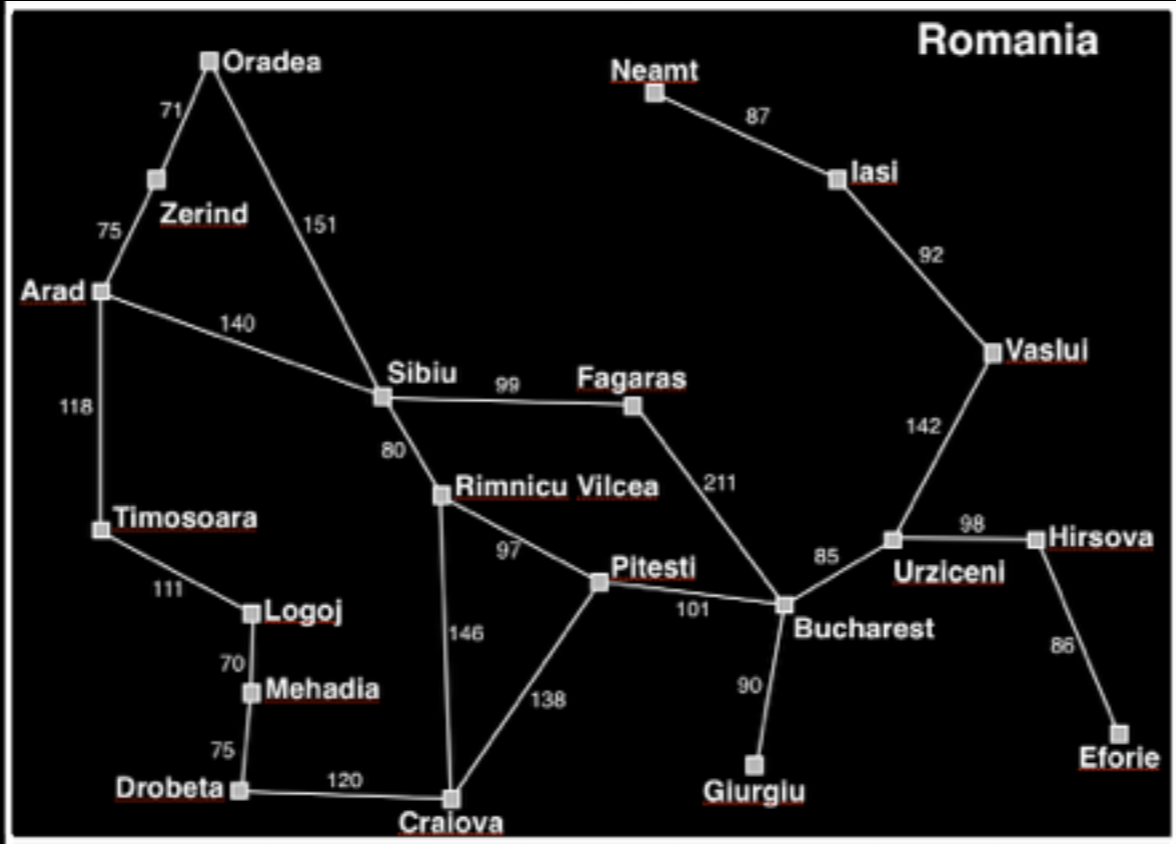
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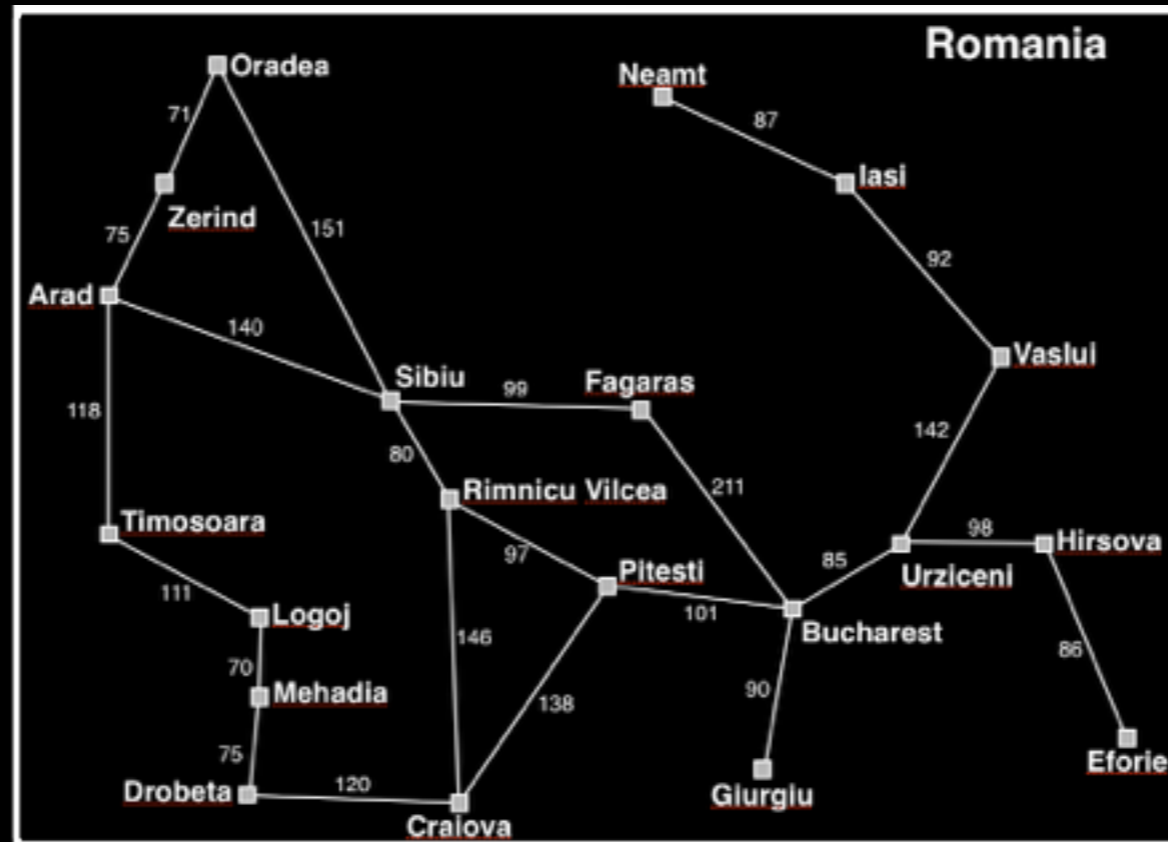
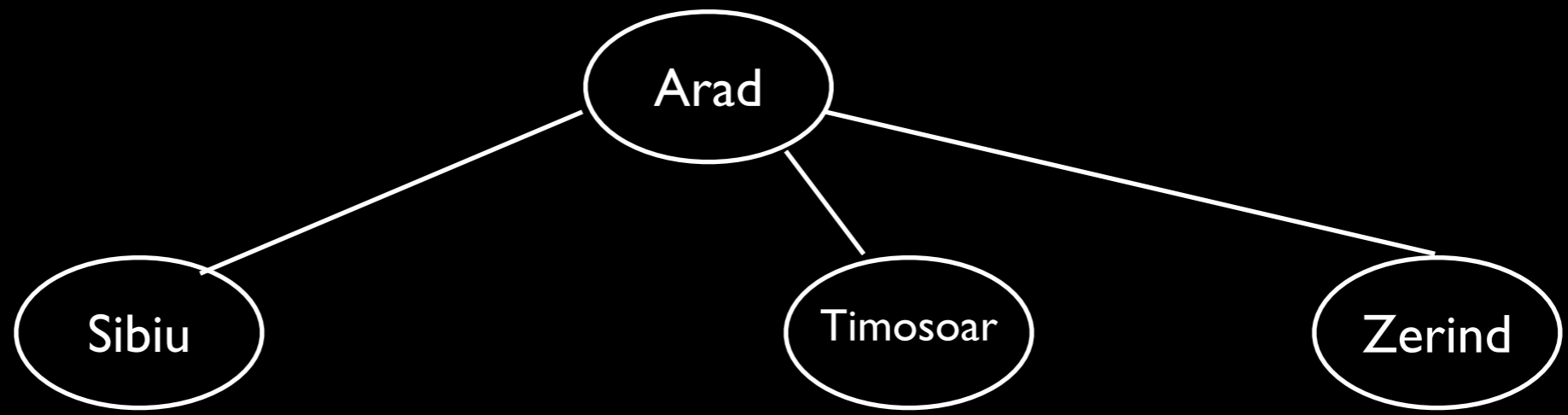


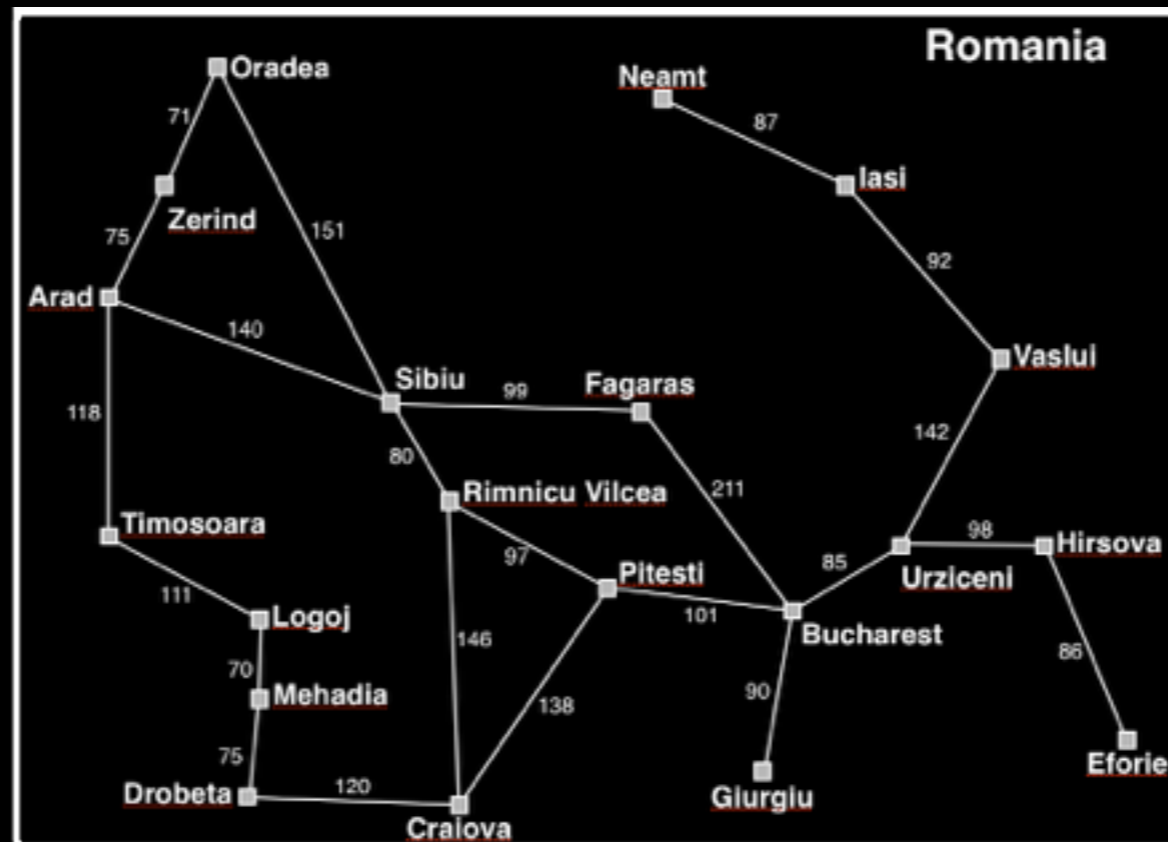
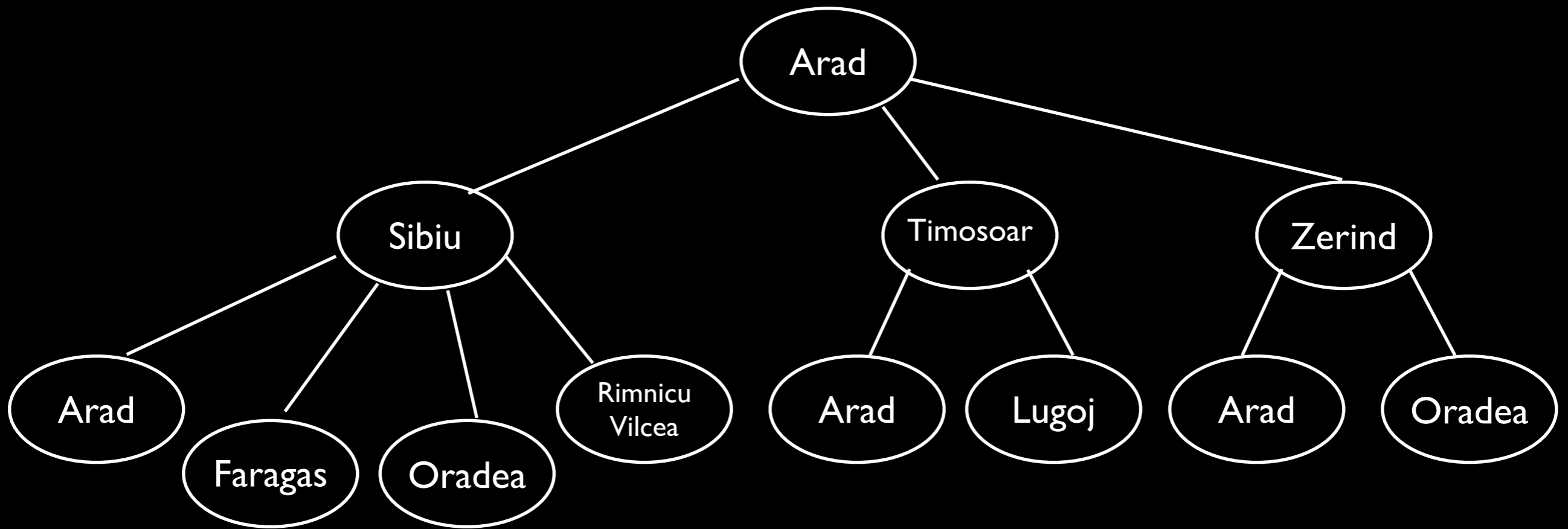
Romania

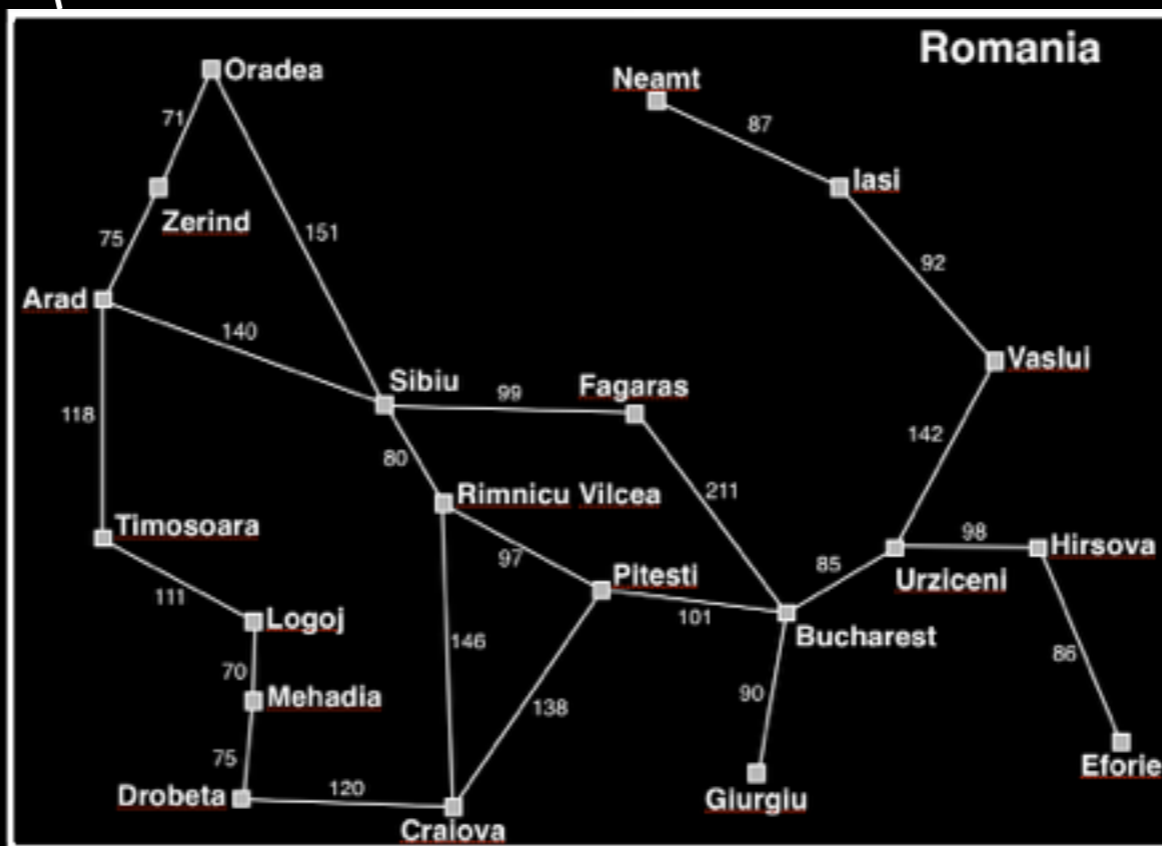
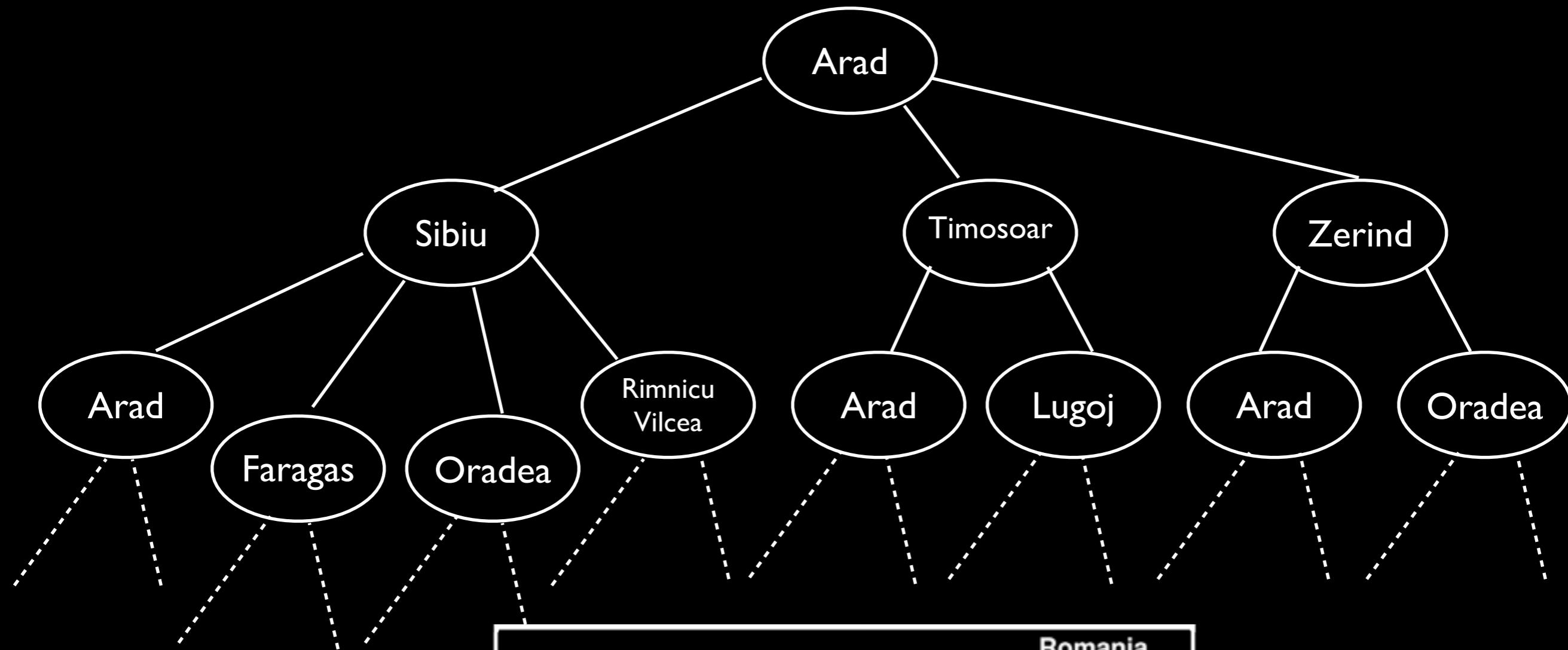


Arad

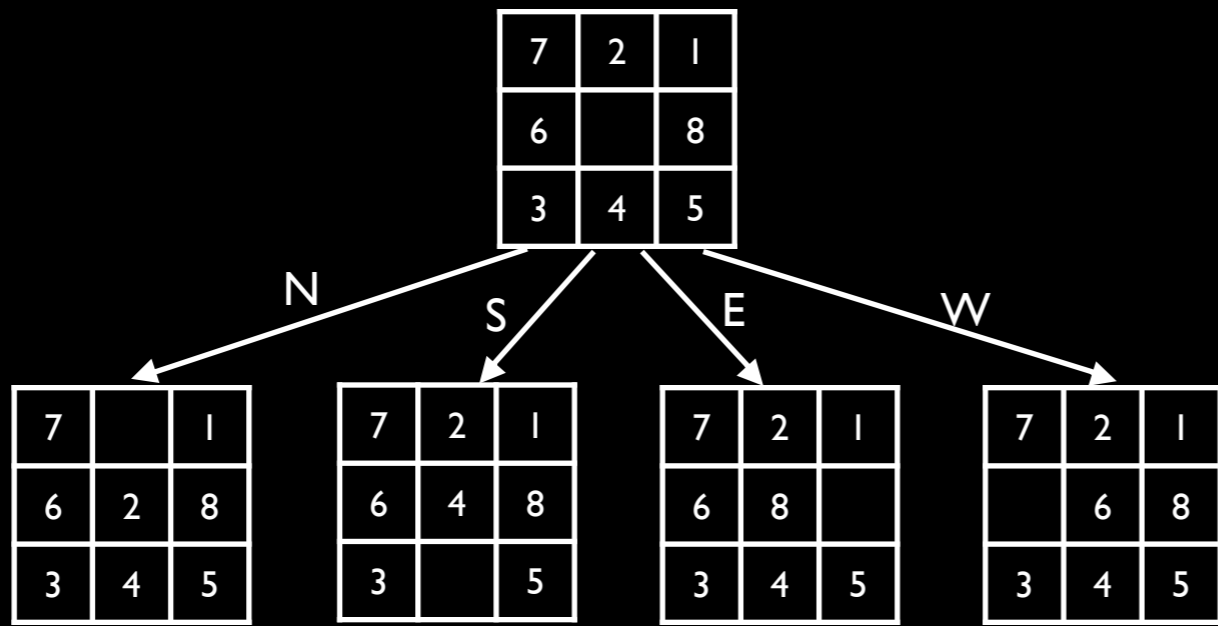


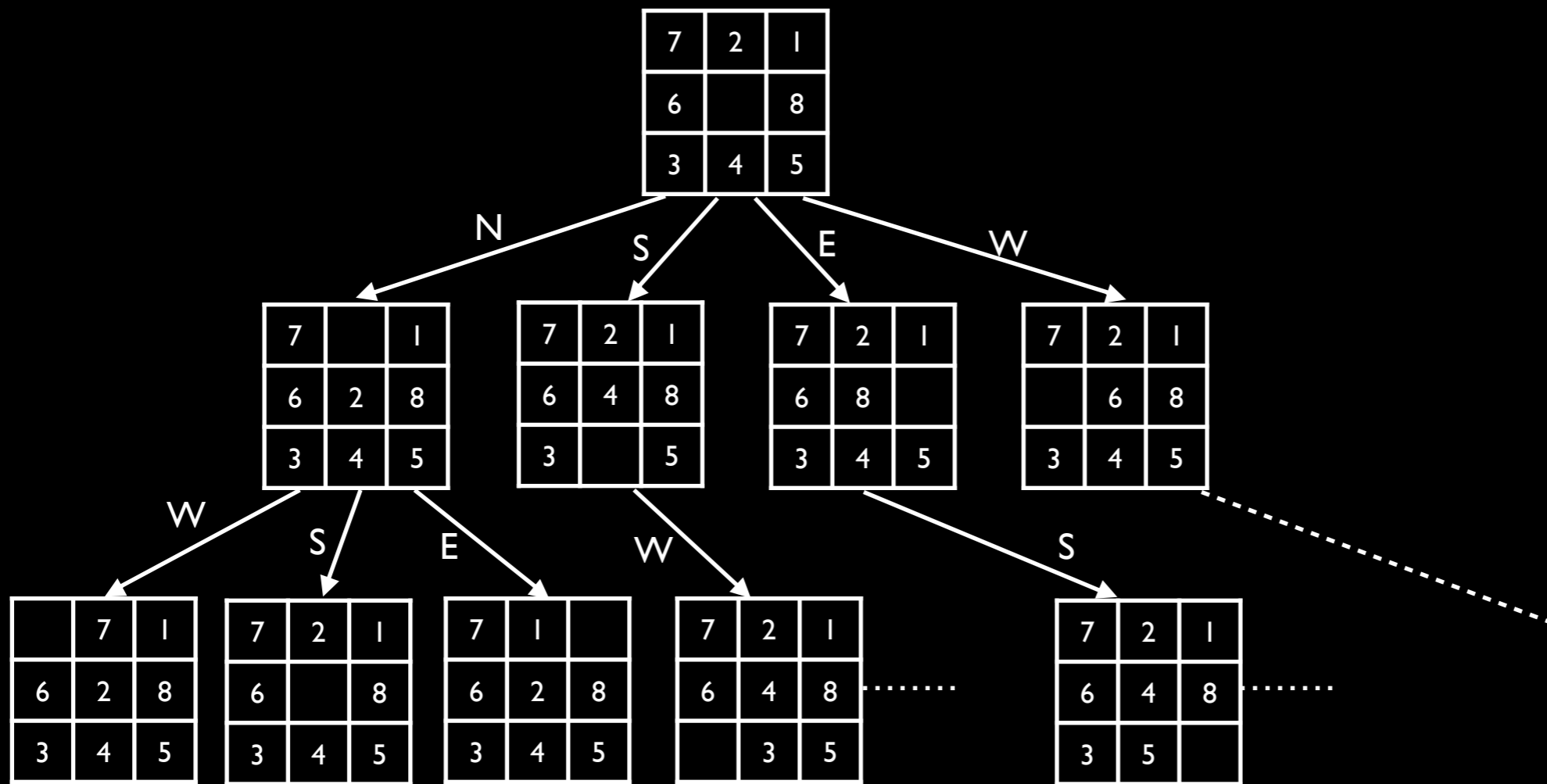


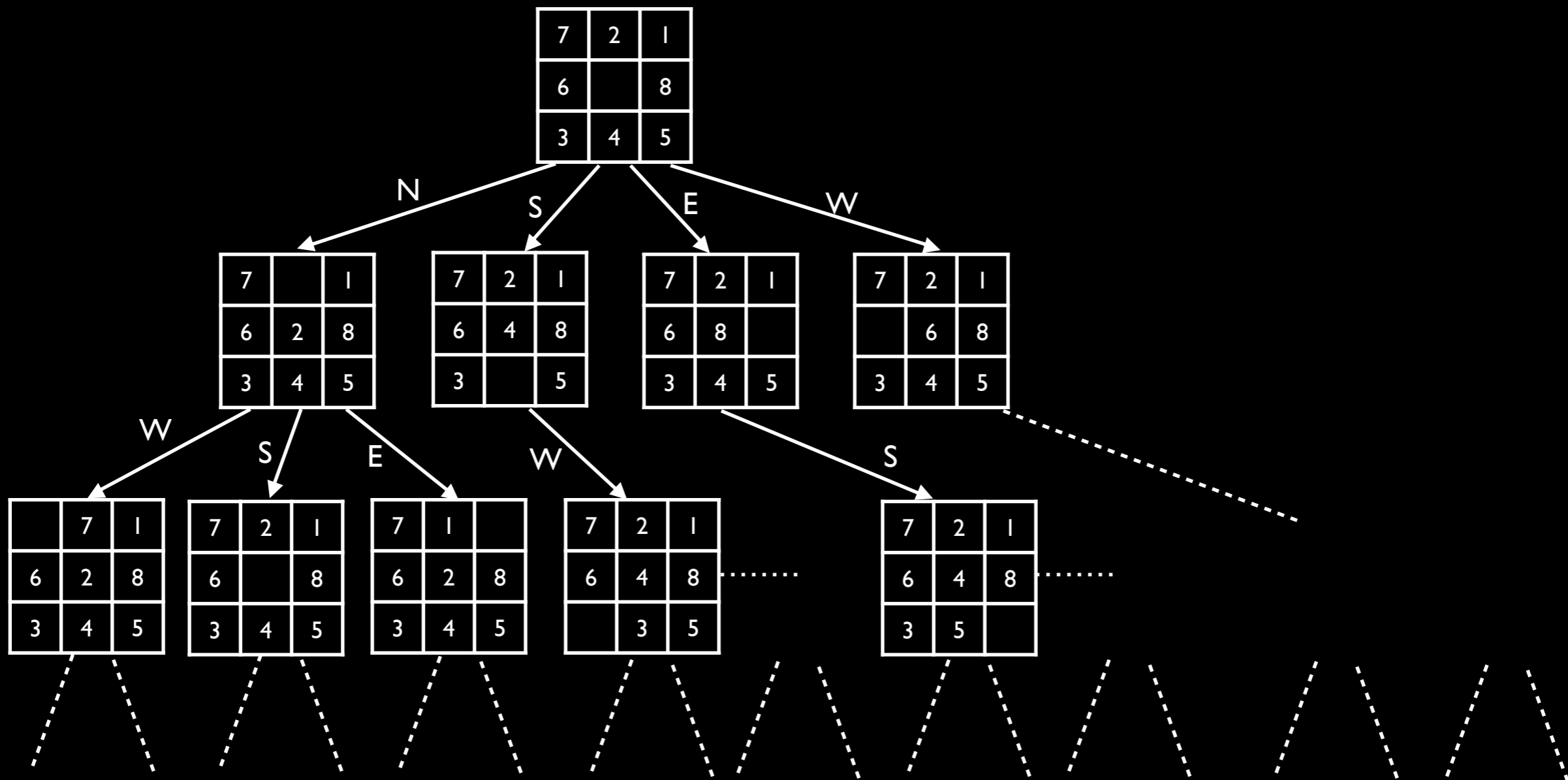




7	2	1
6		8
3	4	5







State-Space Search

- Start with initial state
- Generate successor states by applying applicable actions
- Until you find a goal state

```
Solution treeSearch(Problem p) {
    Set<Node> frontier = new Set<Node>(p.getInitialState());

    while (true) {
        if (frontier.isEmpty()) {
            return null;
        }
        Node node = frontier.selectOne();
        if (p.isGoalState(node.getState())) {
            return n.getSolution();
        }

        for (Node n : node.expand()) {

            frontier.add(n);

        }
    }
}
```

```
Solution graphSearch(Problem p) {
    Set<Node> frontier = new Set<Node>(p.getInitialState());
    Set<Node> explored = new Set<Node>();
    while (true) {
        if (frontier.isEmpty()) {
            return null;
        }
        Node node = frontier.selectOne();
        if (p.isGoalState(node.getState())) {
            return n.getSolution();
        }
        explored.add(node);
        for (Node n : node.expand()) {
            if (!explored.contains(n)) {
                frontier.add(n);
            }
        }
    }
}
```

Summary

- General-purpose algorithm for solving any problem that can be represented using states and actions that transition between them
- State-space search framework will allow us to explore and compare alternatives

```
Solution graphSearch(Problem p) {
    Set<Node> frontier = new Set<Node>(p.getInitialState());
    Set<Node> explored = new Set<Node>();
    while (true) {
        if (frontier.isEmpty()) {
            return null;
        }
        Node node = frontier.selectOne();
        if (p.isGoalState(node.getState())) {
            return n.getSolution();
        }
        explored.add(node);
        for (Node n : node.expand()) {
            if (!explored.contains(n)) {
                frontier.add(n);
            }
        }
    }
}
```

Next Class

Your Homework this
Weekend:

Read Chapters 2 and 3