# CSC242: Intro to AI

Lecture 3

Search Strategies

```
Solution graphSearch(Problem p) {
  Set<Node> frontier = new Set<Node>(p.getInitialState());
  Set<Node> explored = new Set<Node>();
  while (true) {
    if (frontier.isEmpty()) {
      return false;
    }
    Node node = frontier.selectOne();
    if (p.isGoalState(node.getState())) {
      return node.getSolution();
    }
    explored.add(node);
    for (Node n : node.expand()) {
      if (!explored.contains(n)) {
        frontier.add(n);
      }
    }
  }
}
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# Kinds of Strategies

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- Uninformed
  - Only considers the structure of the search space given by the transition function
  - Does not consider the state information associated with a node

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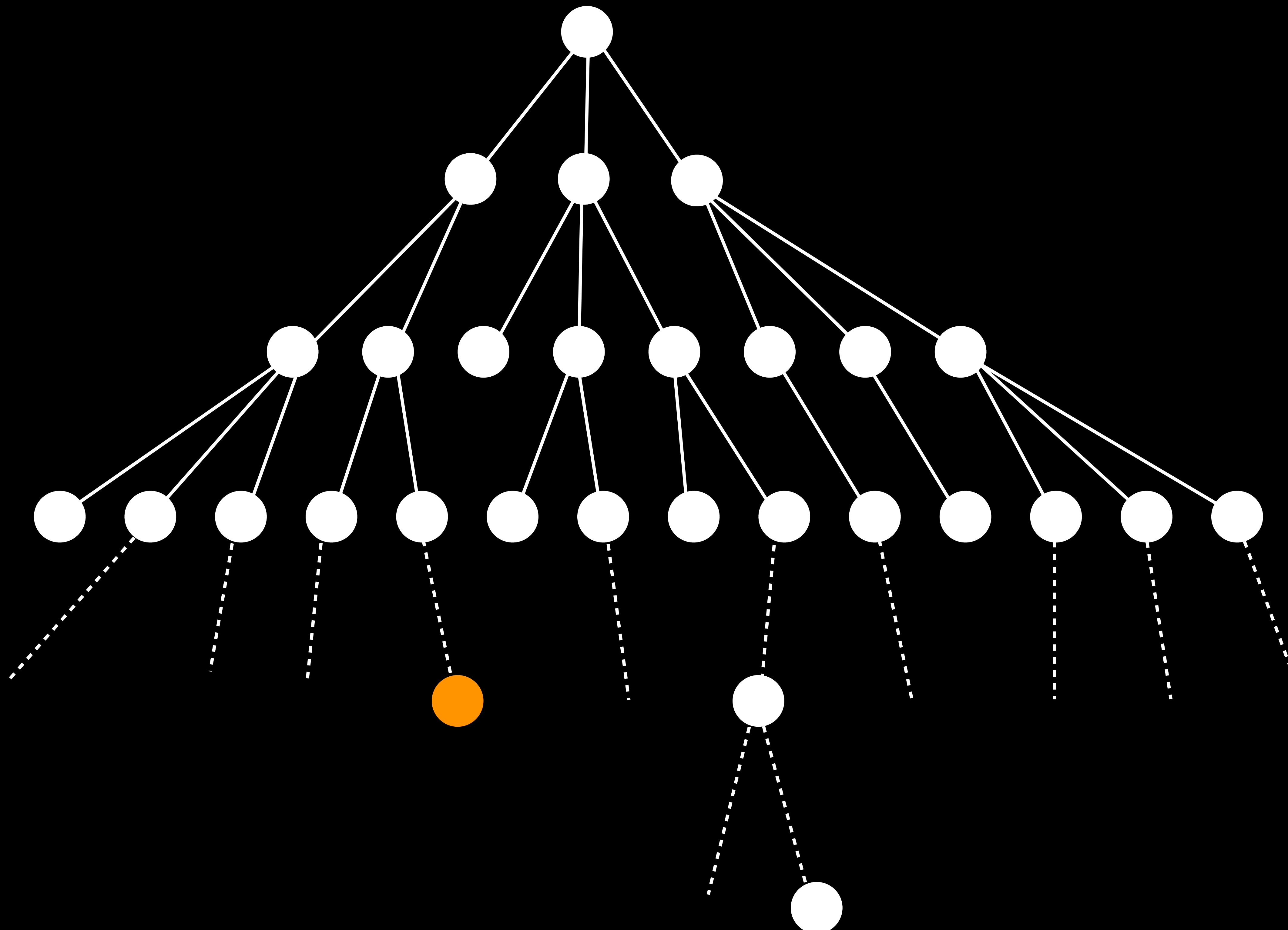
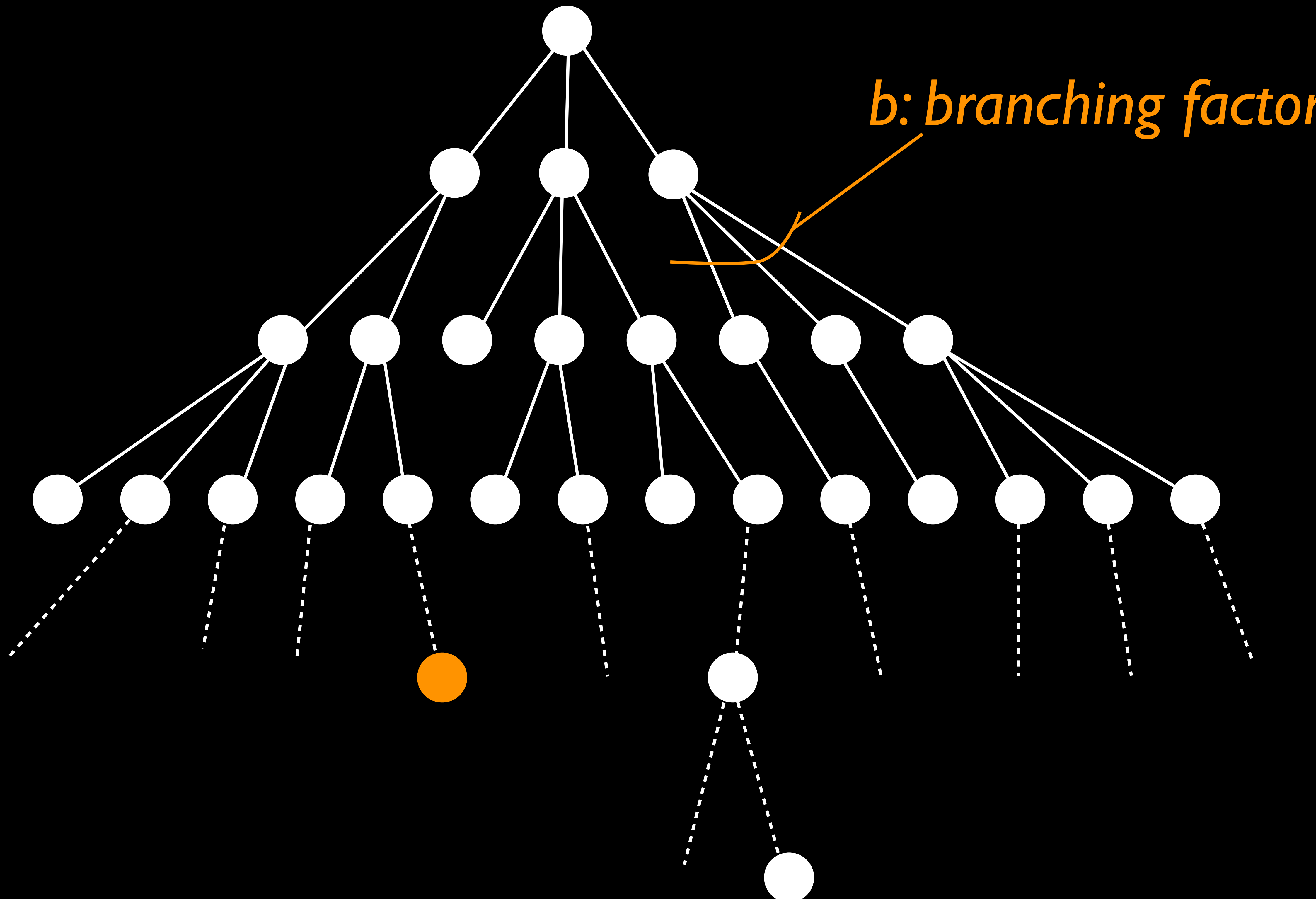
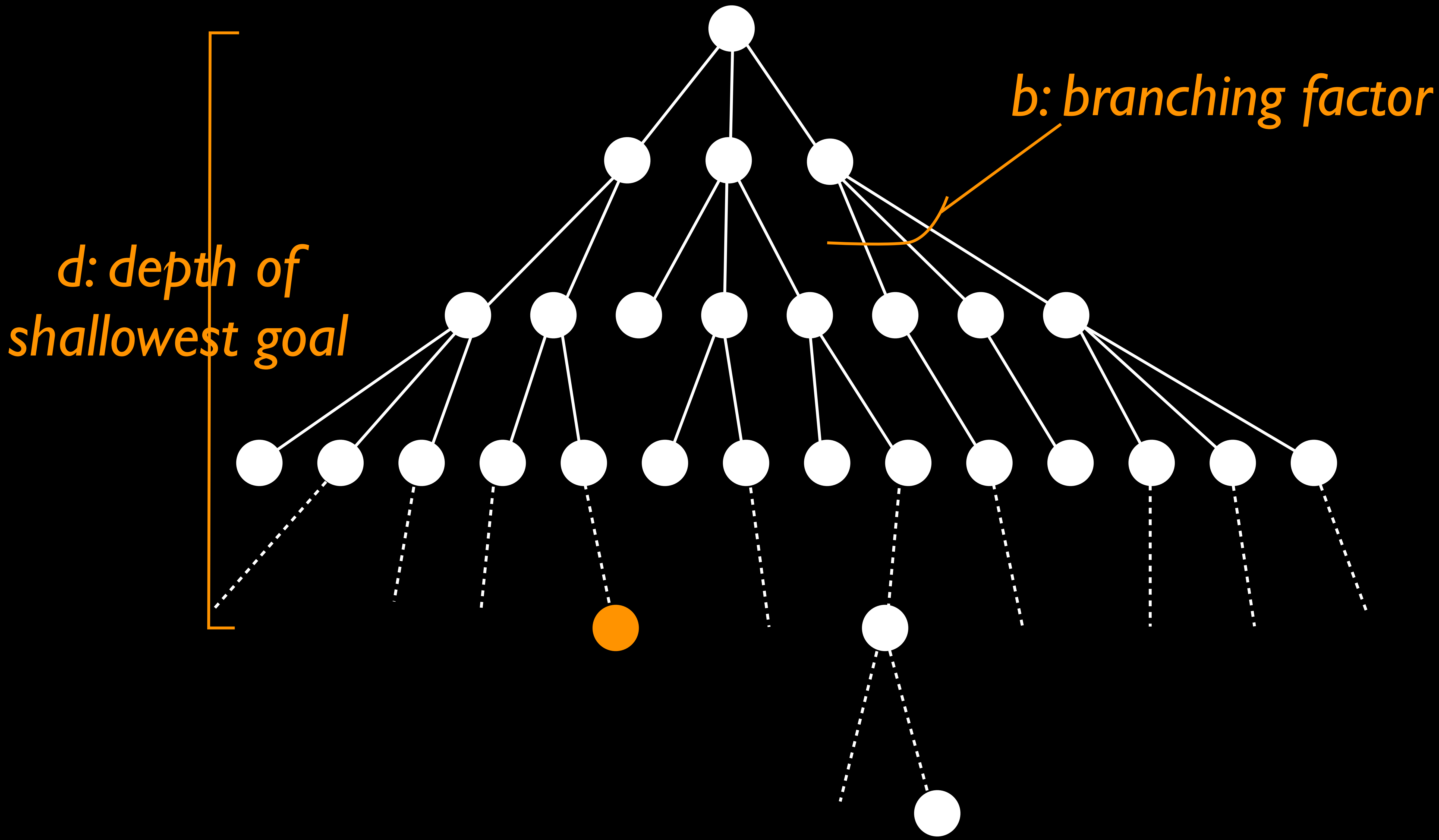
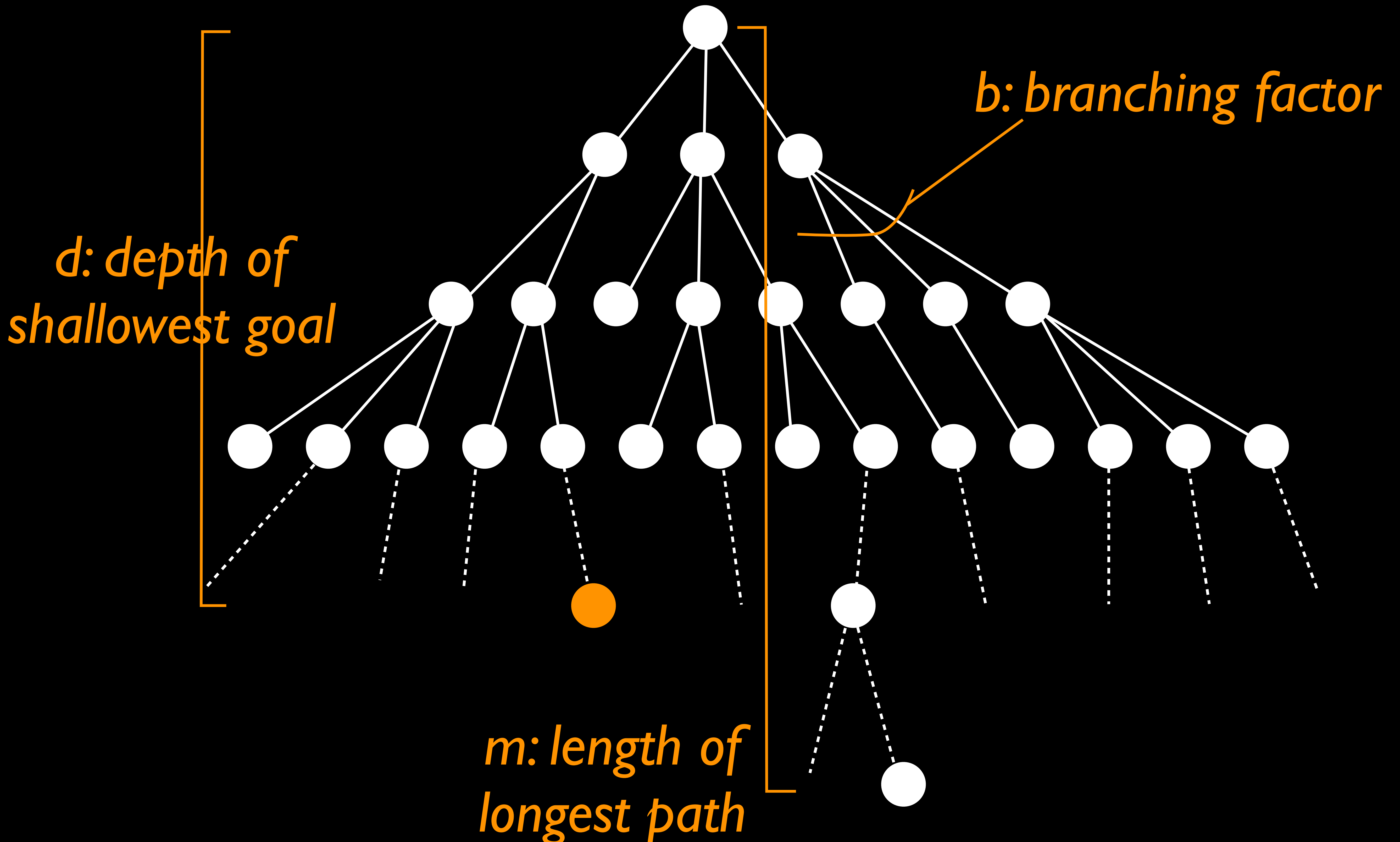
- Uninformed
  - Only considers the structure of the search space given by the transition function
  - Does not consider the state information associated with a node
- Heuristic
  - Uses the state information to help select promising nodes to expand




b: branching factor


b: branching factor
d: depth of
shallowest goal


b: branching factor
d: depth of shallowest goal
m: length of longest path

# Uninformed Strategies

# Uninformed Strategies

Credit: Ben Hider/Getty Images

# $100

# $100

- A kind of uninformed search that
  - Is guaranteed to find a goal state if one exists, and the branching factor is finite
  - Is guaranteed to find a goal that has smallest depth

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- A kind of uninformed search that
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- What is breadth-first search?

# $200

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- A data structure used to represent the frontier in breadth-first search that allows "select_one" to be performed in constant time.

# $200

- A data structure used to represent the frontier in breadth-first search that allows “select_one” to be performed in constant time.
- What is a FIFO (first-in first-out) queue?

# $300

# $300

- The time complexity of breadth first-search, where b is the maximum branching factor and d is the depth of the shallowest goal

# $300

- The time complexity of breadth first-search, where b is the maximum branching factor and d is the depth of the shallowest goal
- What is $O(b^d)$?

# $400

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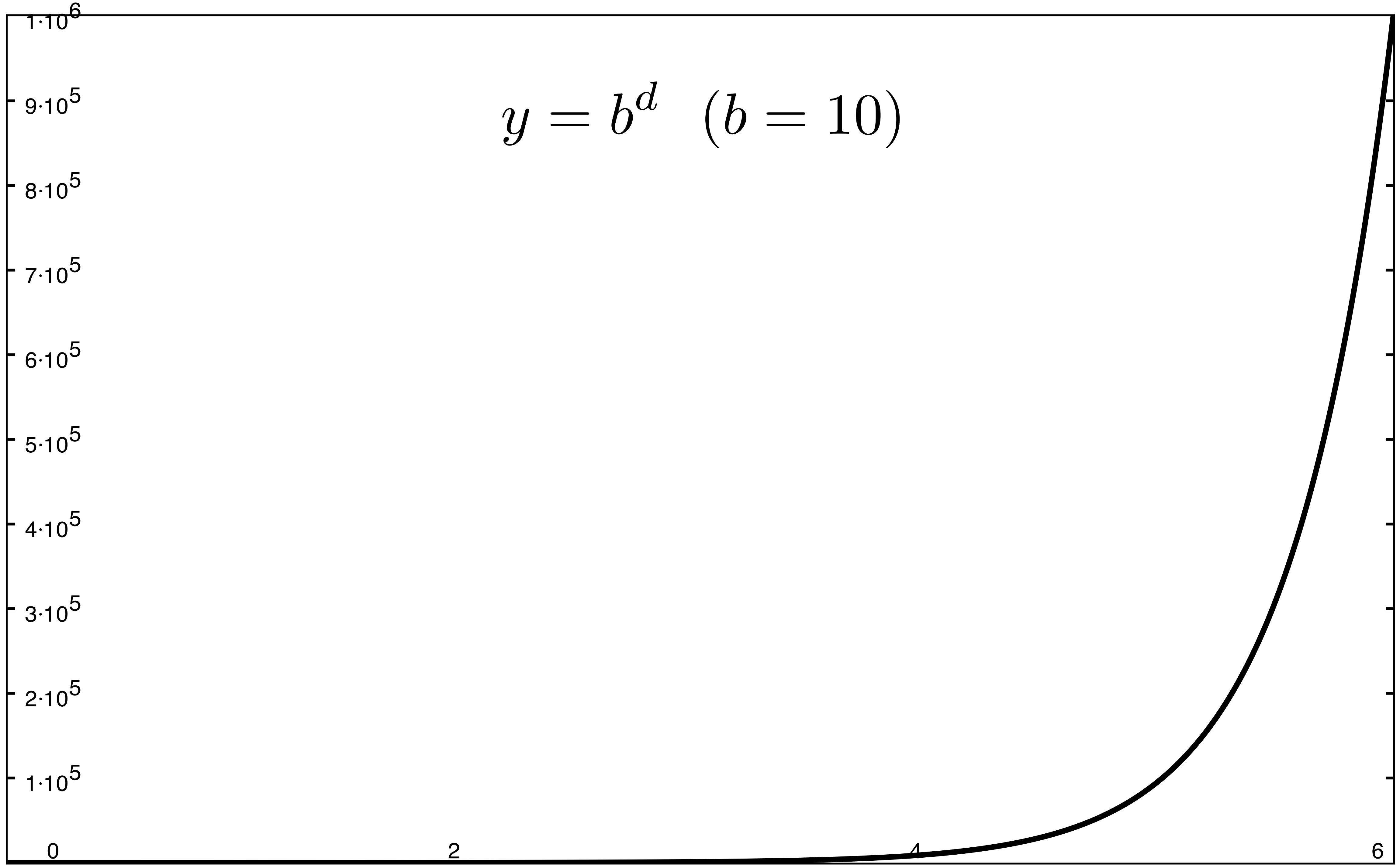
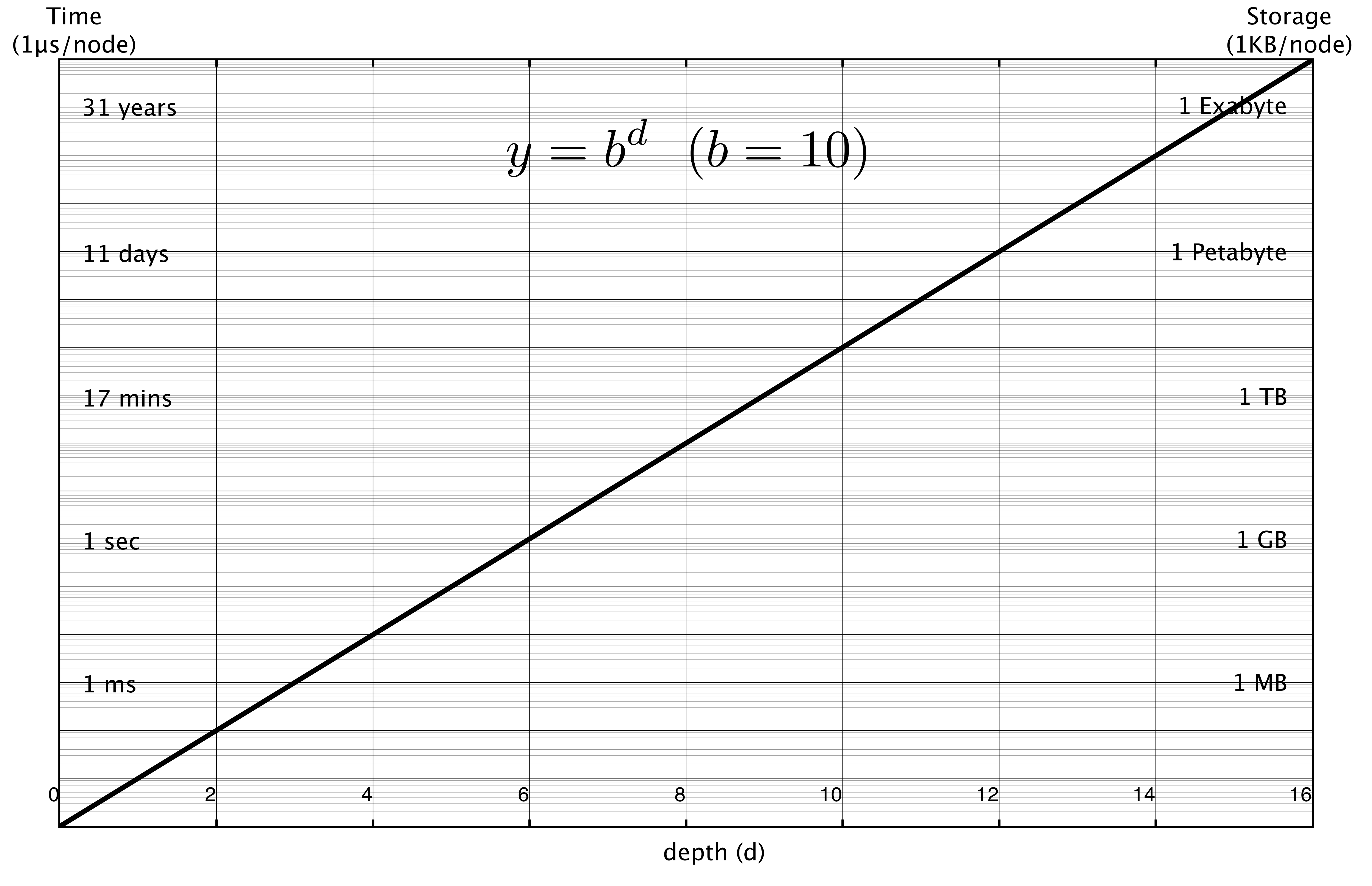
- If the state space has this property, then breadth-first search may fail to find a solution, even when one exists
- What is having an infinite branching factor?



$y = b^d \quad (b = 10)$
$1{\cdot}10^6$
$9{\cdot}10^5$
$8{\cdot}10^5$
$7{\cdot}10^5$
$6{\cdot}10^5$
$5{\cdot}10^5$
$4{\cdot}10^5$
$3{\cdot}10^5$
$2{\cdot}10^5$
$1{\cdot}10^5$
0
2
4
6



Time
(1μs/node)
Storage
(1KB/node)
31 years
1 Exabyte
$y = b^d \quad (b = 10)$
11 days
1 Petabyte
17 mins
1 TB
1 sec
1 GB
1 ms
1 MB
0
2
4
6
8
10
12
14
16
depth (d)

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  - The maximum size of the frontier is mb, where b is the maximum branching factor

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- A data structure used to represent the frontier in depth-first search that allows "select_one" to be performed in constant time.
- What is a stack (or what is a LIFO last-in first-out queue)?

# $300

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- The time complexity of depth first-search, where b is the branching factor, d is the depth of the shallowest goal, and m is the maximum depth of node

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- The time complexity of depth first-search, where b is the branching factor, d is the depth of the shallowest goal, and m is the maximum depth of node
- What is $O(b^m)$?

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- You are usually off performing tree depth first search instead of graph depth first search because this data structure can grow to size $b^m$

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- You are usually off performing tree depth first search instead of graph depth first search because this data structure can grow to size $b^m$
- What is the explored list?

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    for (Node n : node.expand()) {

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- This simple trick enables breadth-first search to be complete, even if the maximum depth m is infinite

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- This simple trick enables breadth-first search to be complete, even if the maximum depth m is infinite
- What is iterative deepening?

# Iterative Deepening

# Iterative Deepening

For d = 1, 2, 3, ...

    Do DFS to depth d

Until a goal is found.

| Depth | Number of nodes expanded |
|---|---|
| 1 | $b$ |
| 2 | $b + b^2$ |
| 3 | $b + b^2 + b^3$ |
| $d$ | $b + b^2 + b^3 + \cdots b^d$ |

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$$(d)b + (d-1)b^2 + (d-2)b^3 + \cdots + (1)b^d$$

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$$(d)b + (d-1)b^2 + (d-2)b^3 + \cdots + (1)b^d = O(b^d)$$

# Uninformed Strategies

| | BFS | DFS (tree) | IDS (tree) |
|---|---|---|---|
| Complete? | ✓ | ✗ | ✓ |
| Optimal? | ✓ | ✗ | ✓ |
| Time | $O(b^d)$ | $O(b^m)$ | $O(b^d)$ |
| Space | $O(b^d)$ | $O(bm)$ | $O(bd)$ |

* If step costs are identical (see book)

# Daily Double

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- Replacing the queue in breadth-first search with this data structure enables it find a shortest path to a goal where edges have weights

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- Replacing the queue in breadth-first search with this data structure enables it find a shortest path to a goal where edges have weights
- What is a heap, where the key is the cost of the path from the start node?

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      }
    }
  }
}
```

```
Solution graphSearch(Problem p) {
  Set<Node> frontier = new Heap<Node>(p.getInitialState(), 0);
  Set<Node> explored = new Set<Node>();
  while (true) {
   if (frontier.isEmpty()) {
      return false;
   }
   node = frontier.selectOne();
   pathcost = node.cost;
   if (p.isGoalState(node.getState())) {
      return node.getSolution()
   }
   explored.add(node);
   for ((n, c) in node.expand()) {
      newcost = pathcost + c;
     if (!explored.contains(n) OR newcost < n.cost){
         n.cost = newcost
         if (frontier.contains(n))
            frontier.changekey(n, newcost)
         else frontier.add(n, newcost)
     }
    }
  }
}
```

# AIMA

"Exponential complexity search problems cannot be solved by uninformed methods for any but the smallest instances."

# Heuristic Strategies

heuristic |hyoōˈristik|
adjective

1. Enabling a person to discover or learn something for themselves : *a "hands-on" or interactive heuristic approach to learning*.
2. Computing proceeding to a solution by trial and error or by rules that are only loosely defined.

ORIGIN early 19th cent.: formed irregularly from Greek ***heuriskein 'find'***

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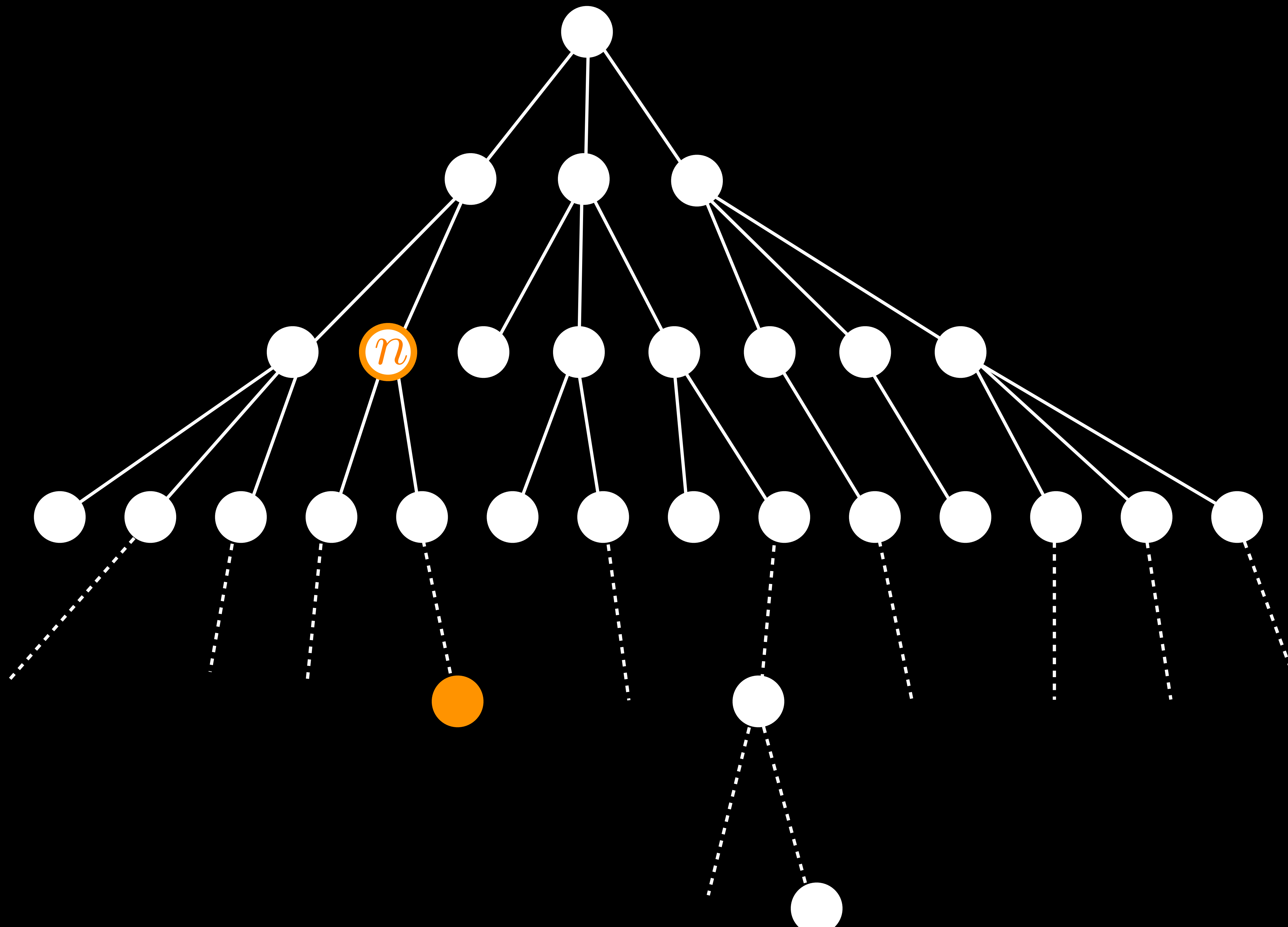
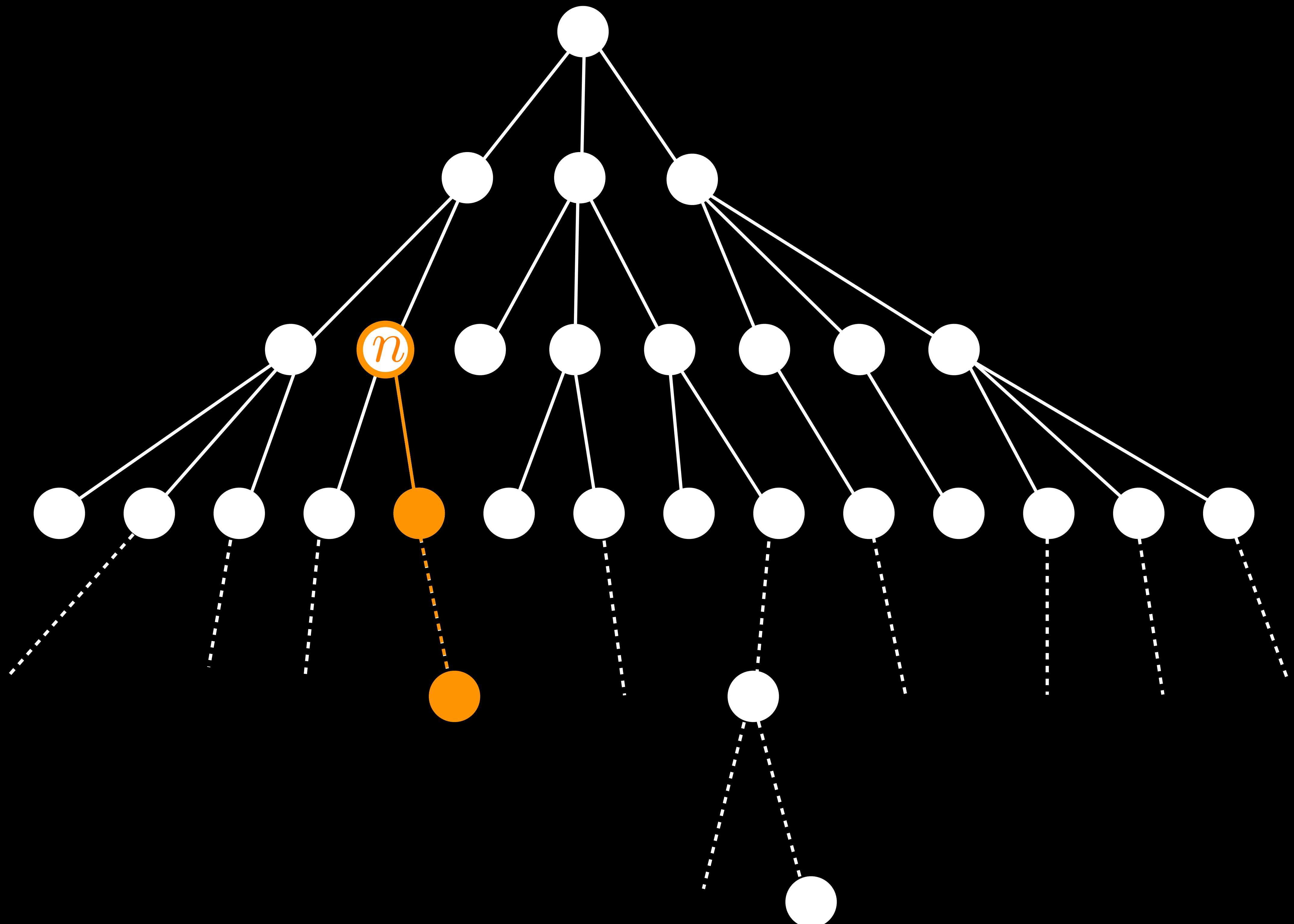
```

# Heuristic Function

$$h(n)$$

Estimated cost of cheapest path from n to a goal node


n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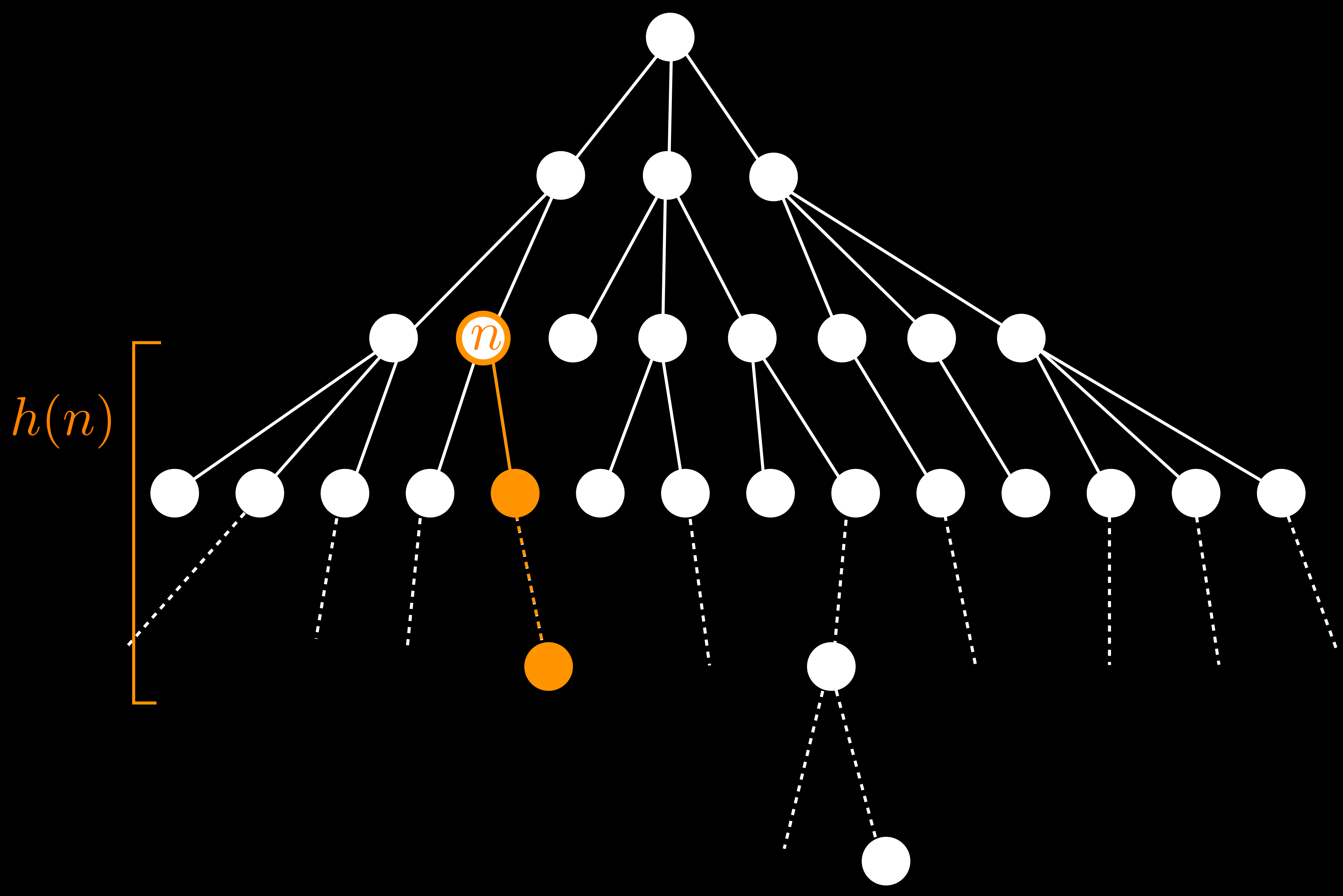

$h(n)$
$n$

Heuristic function

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Estimated cost of cheapest path from *n* to a goal node

Heuristic function

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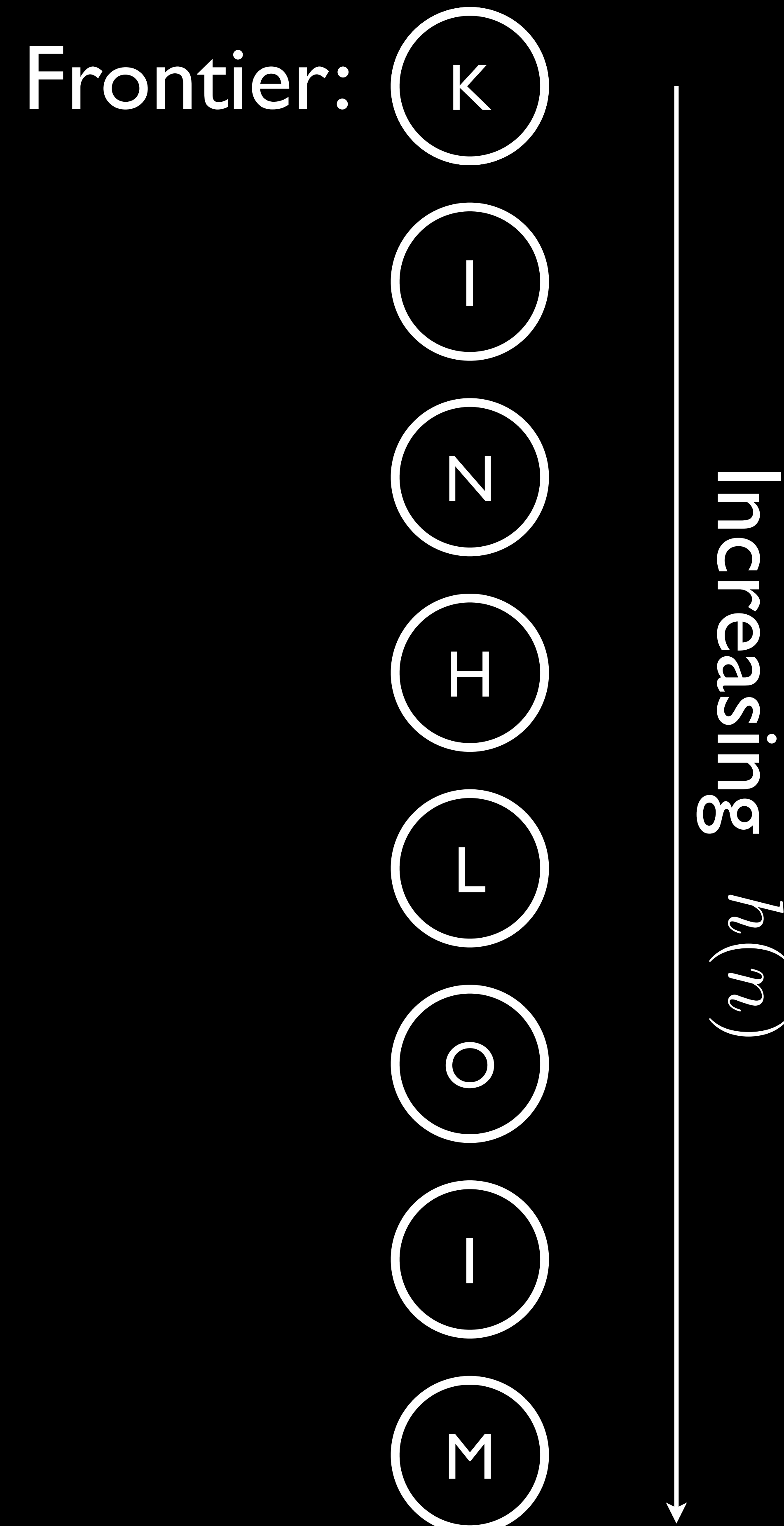

Heuristic function

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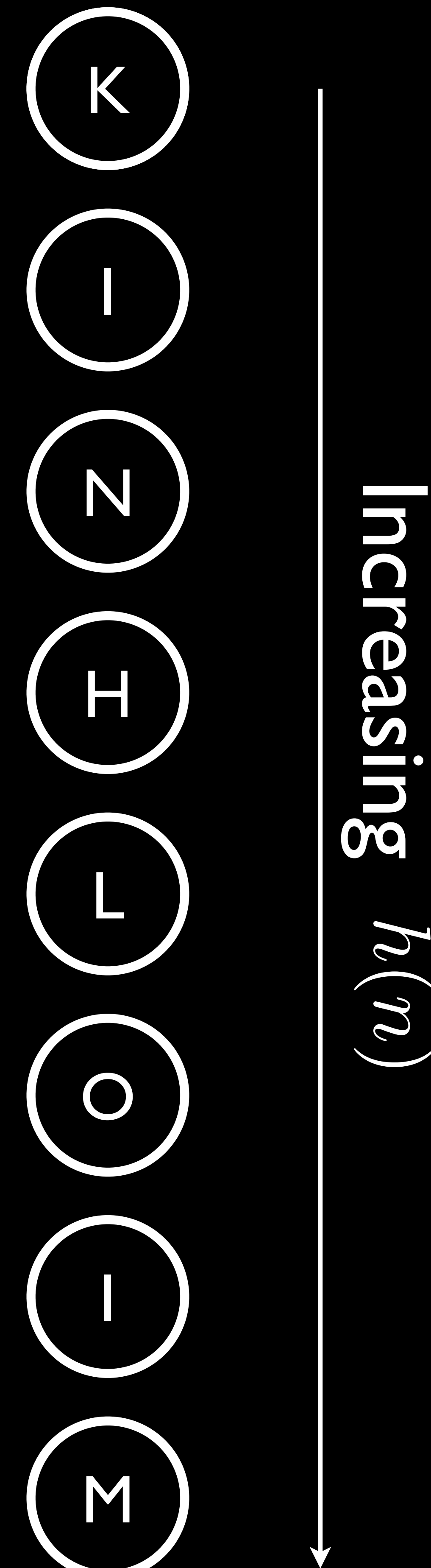
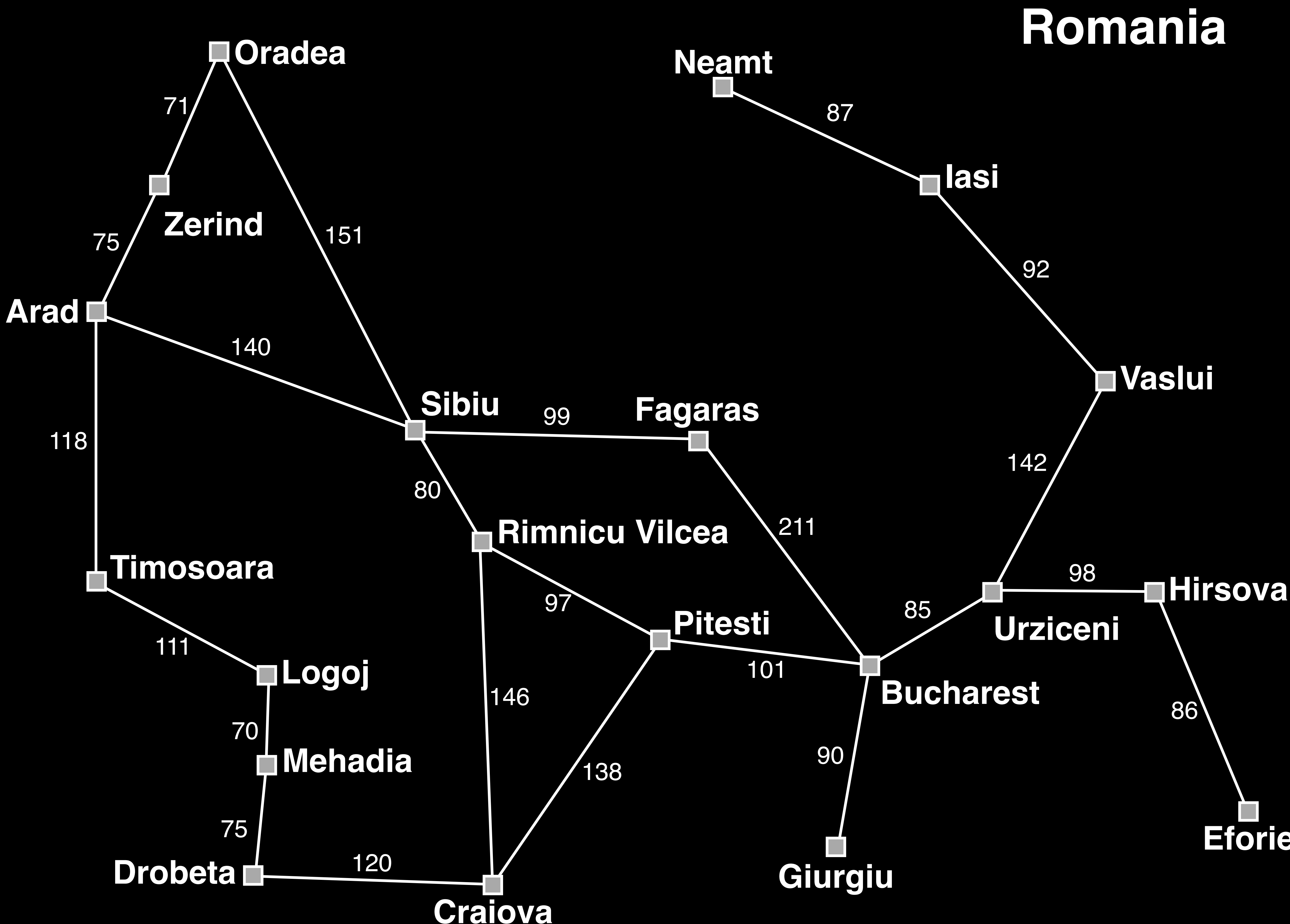
Estimated cost of cheapest path from *n* to a goal node

Frontier:
(Heap)





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Eforie
Drobeta
120
Giurgiu
Craiova

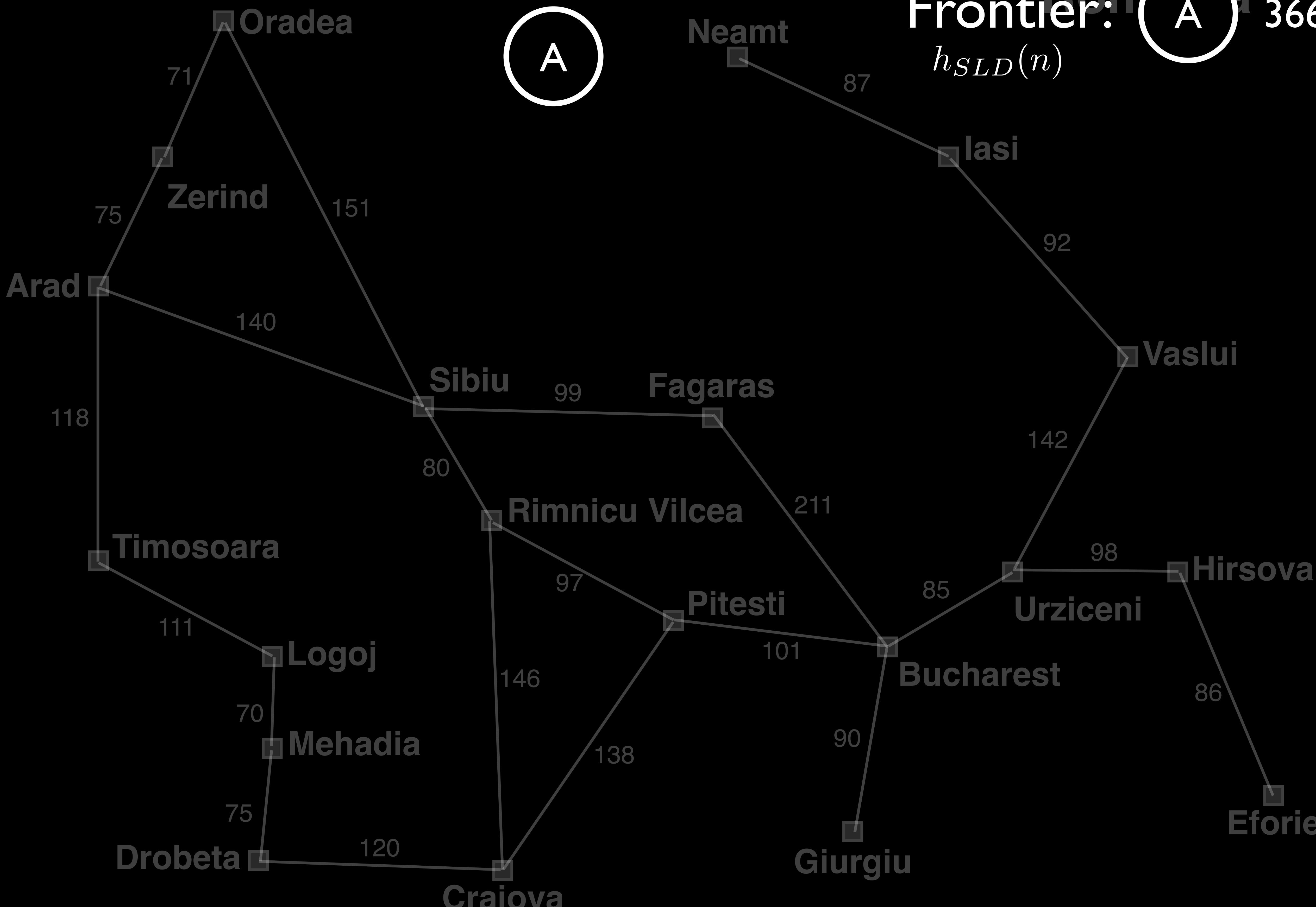
# $h_{SLD}$
## (straight-line distance to Bucharest)

| | |
|---|---|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Drobeta | 242 |
| Eforie | 161 |
| Fagaras | 176 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |

| | |
|---|---|
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 100 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

Frontier: A 366

$h_{SLD}(n)$

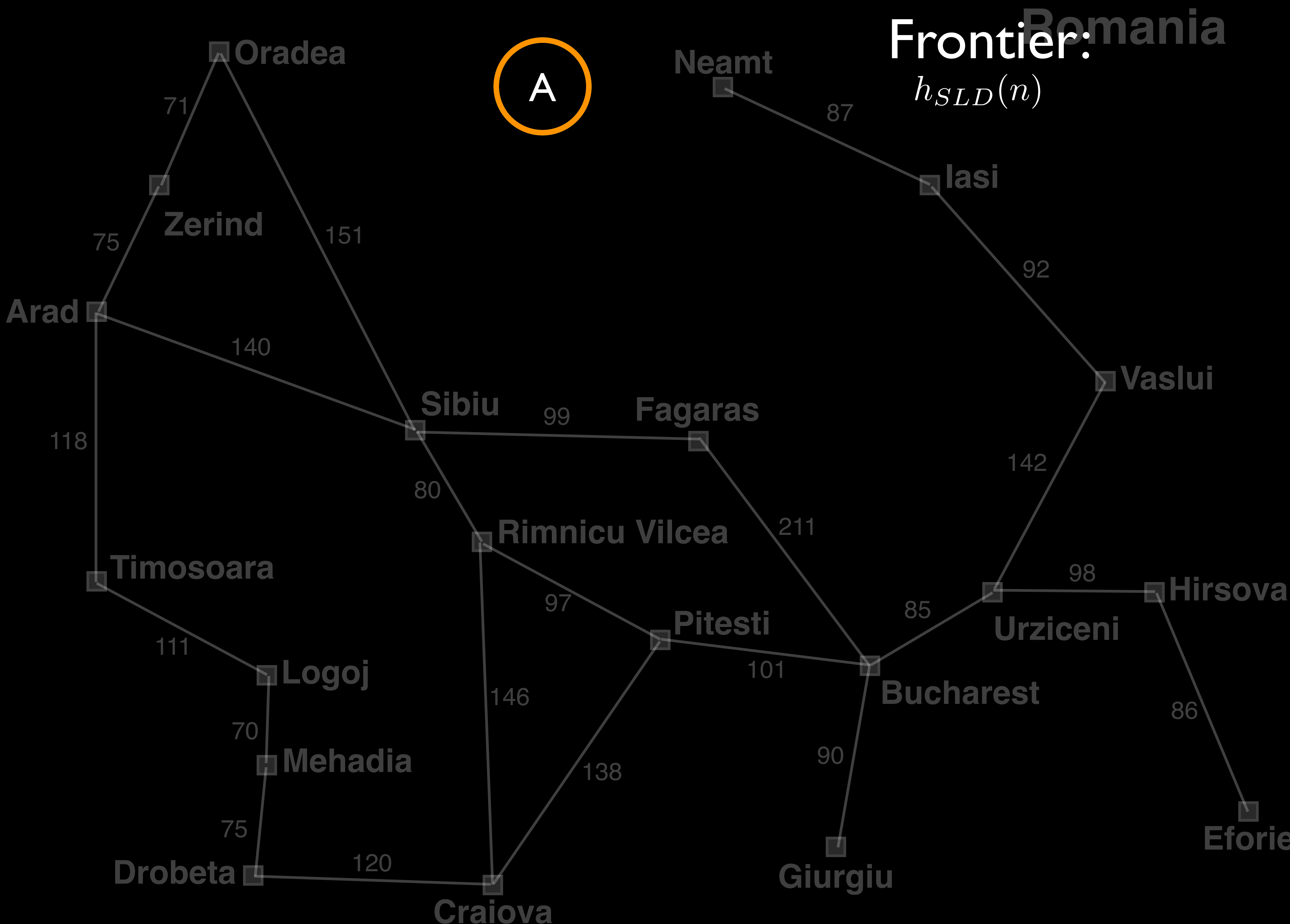


Frontier: A 366

$h_{SLD}(n)$

Frontier:

$h_{SLD}(n)$



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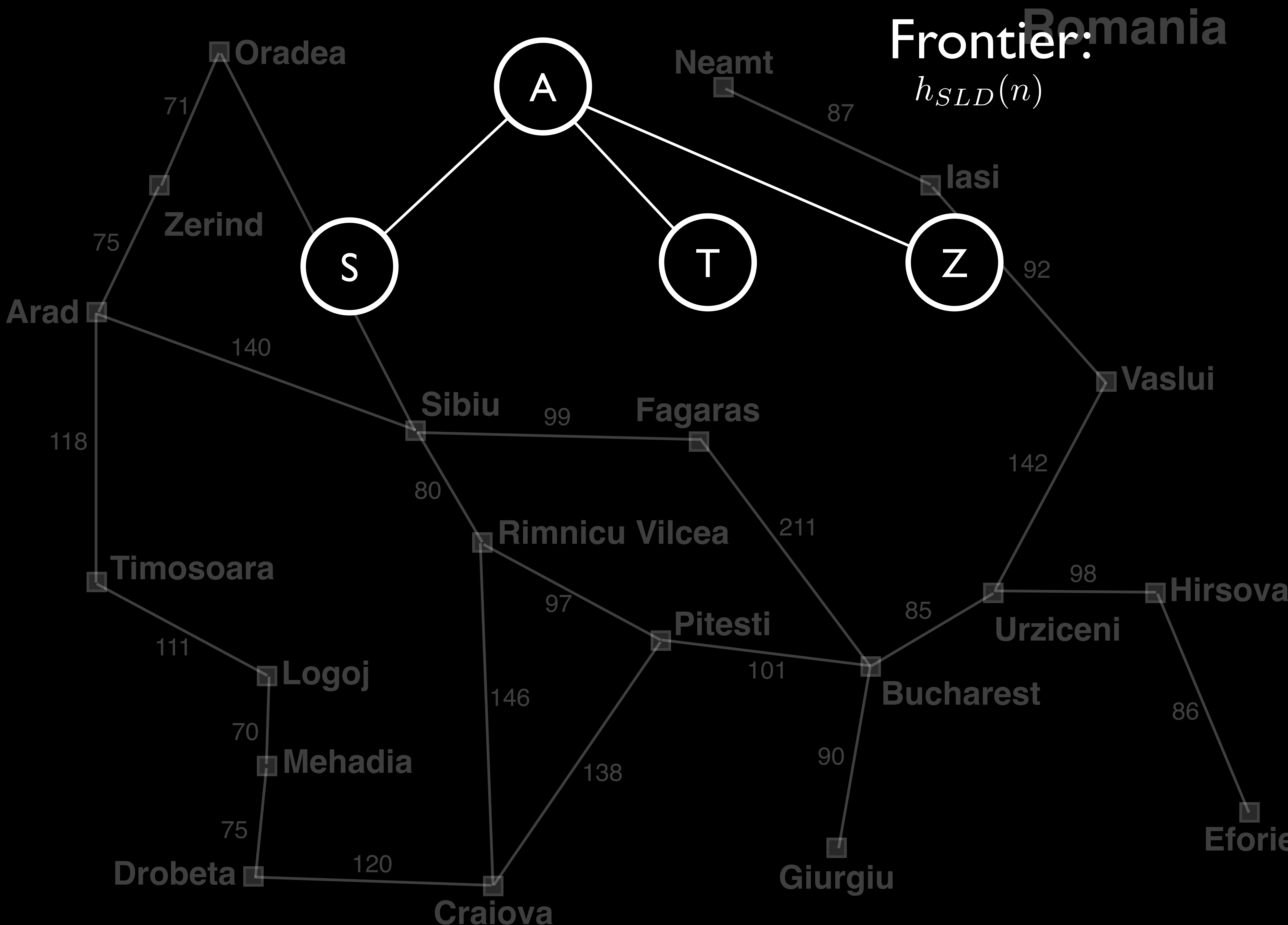

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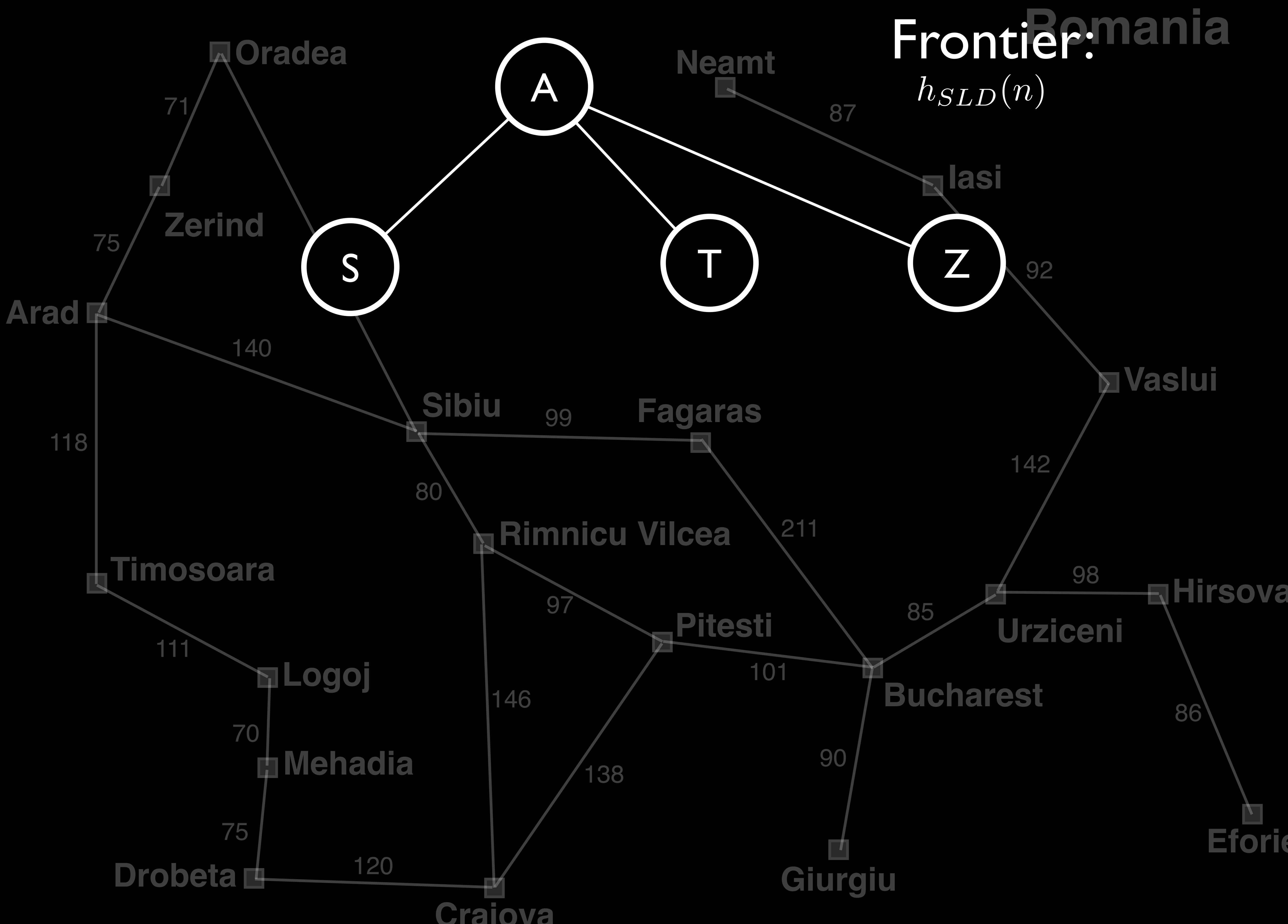

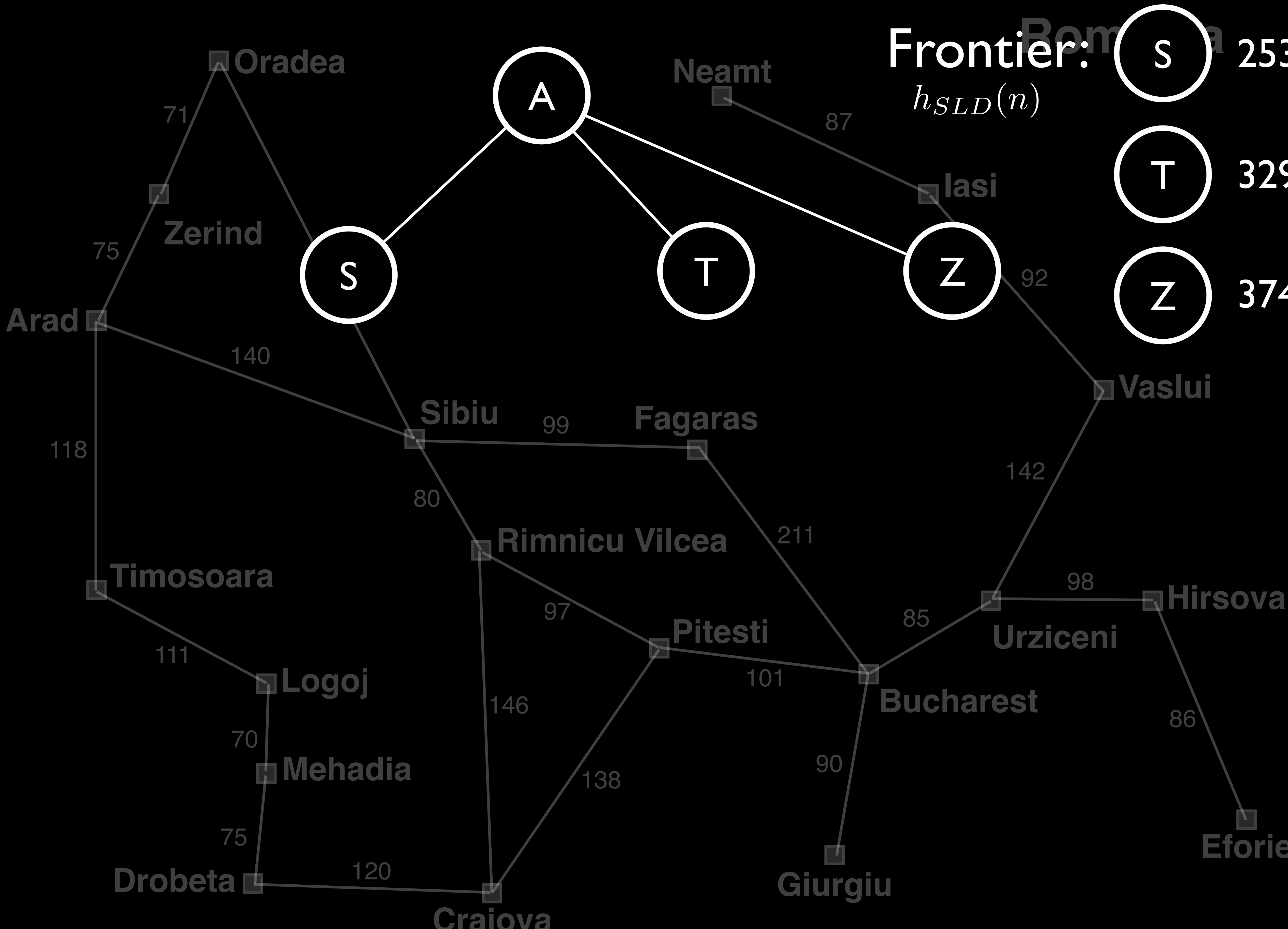

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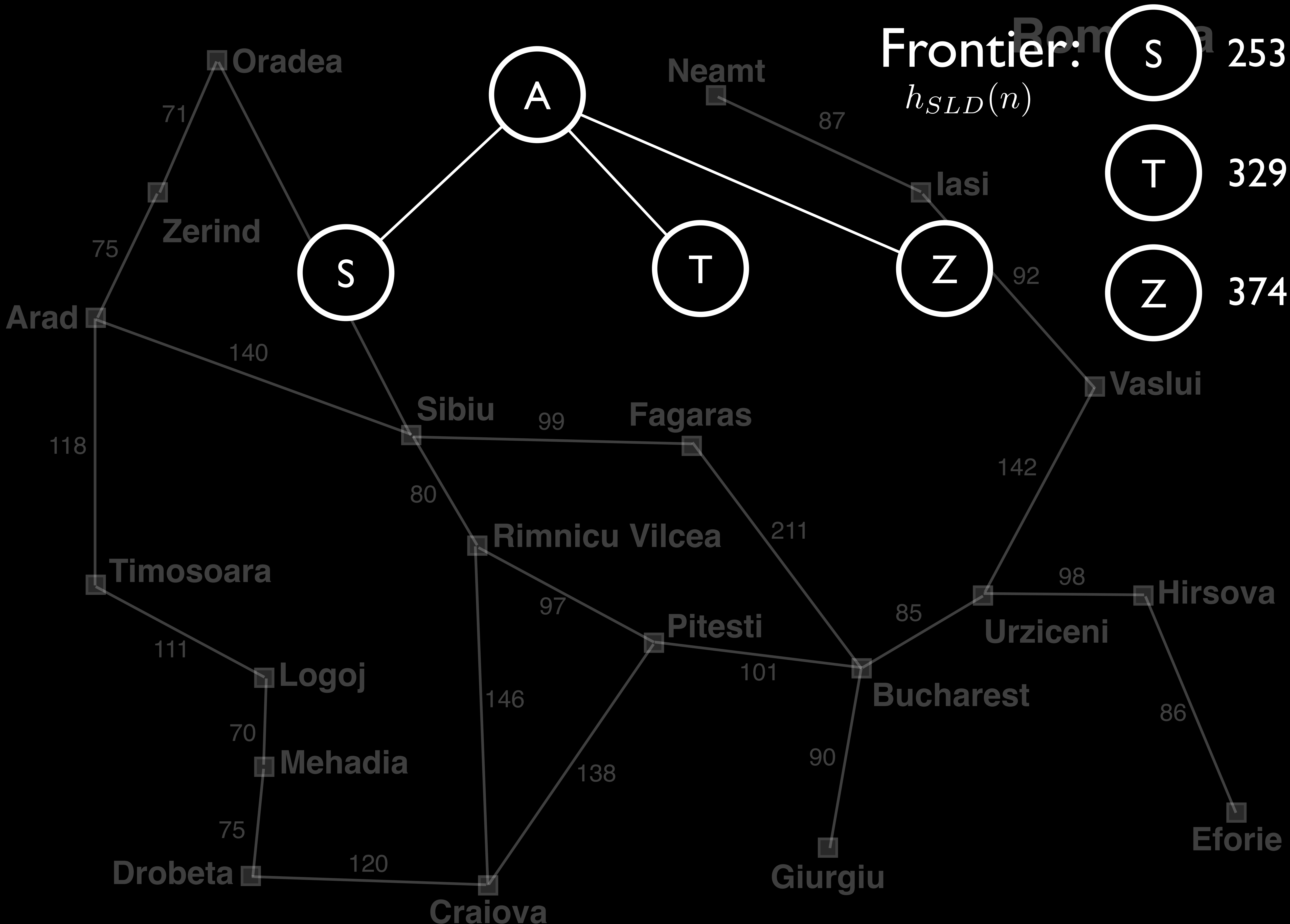

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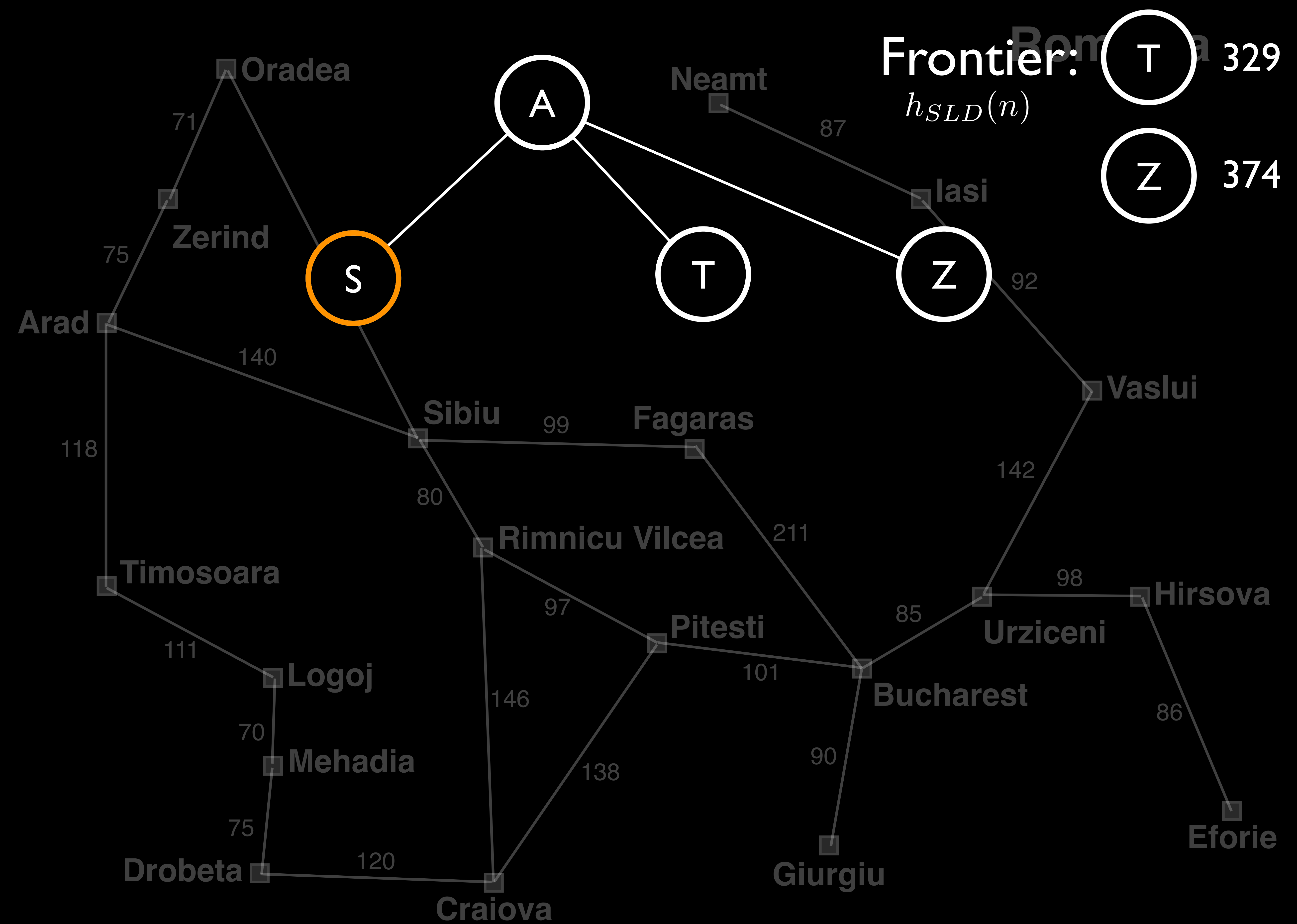
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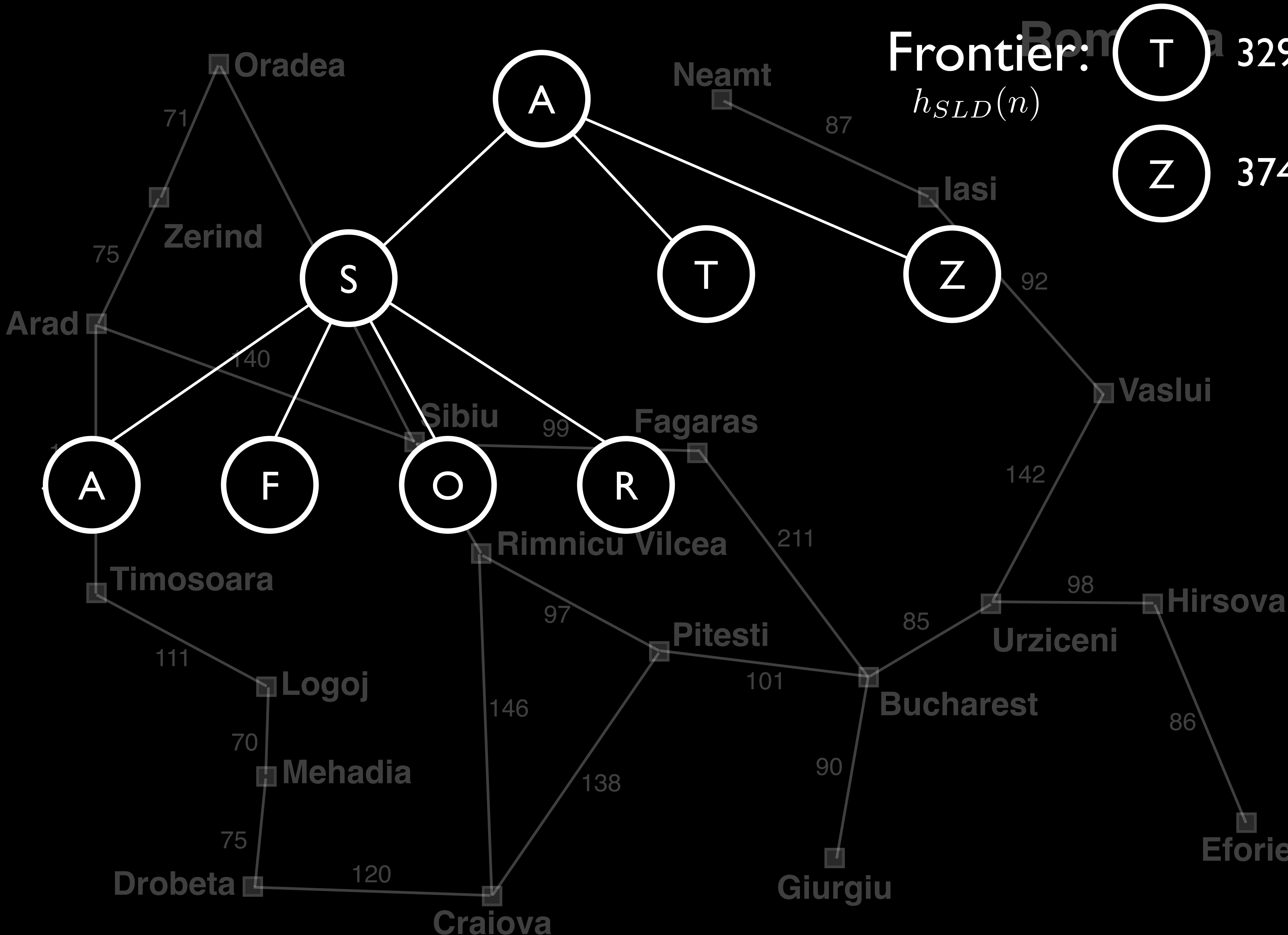

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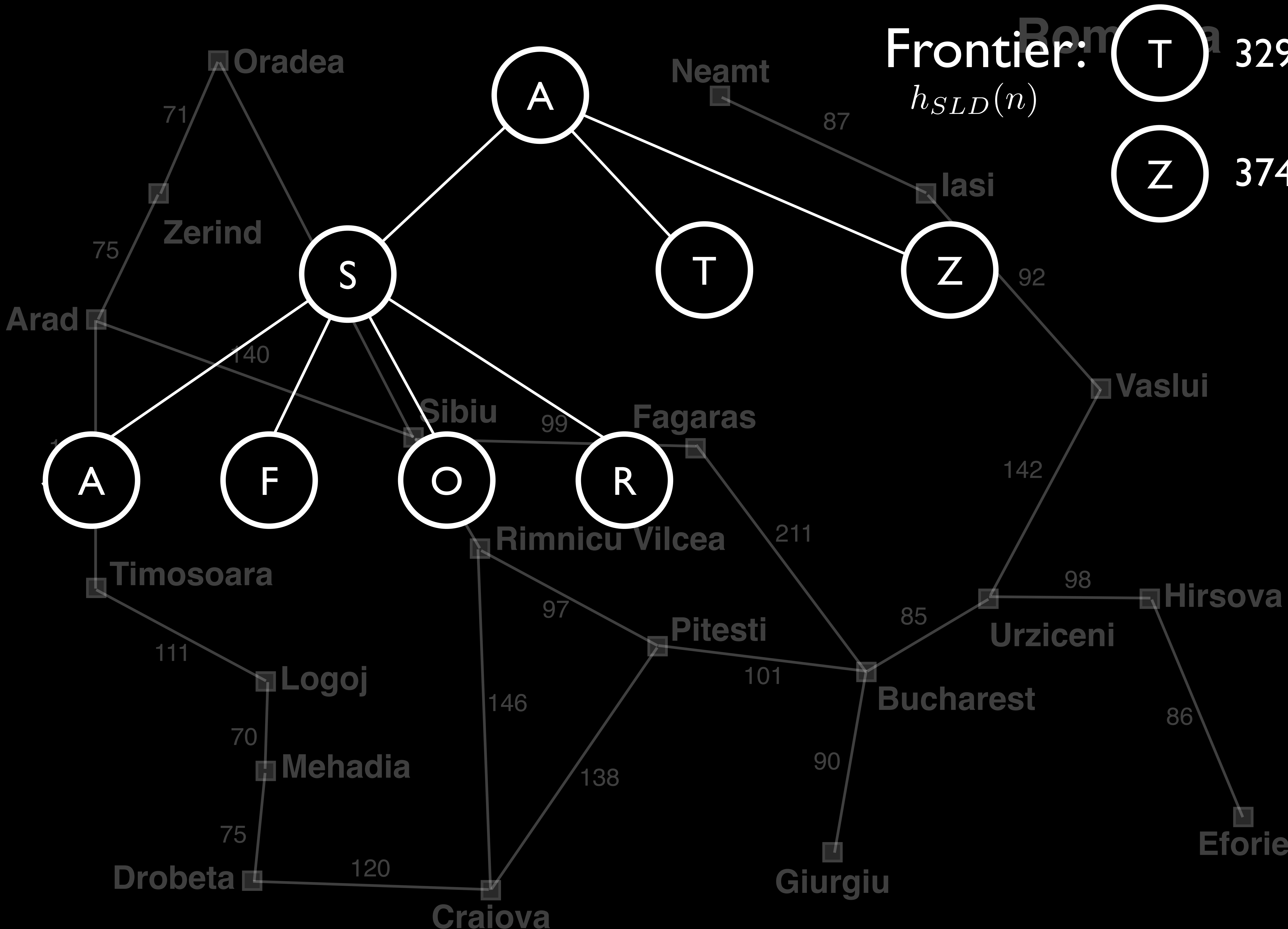
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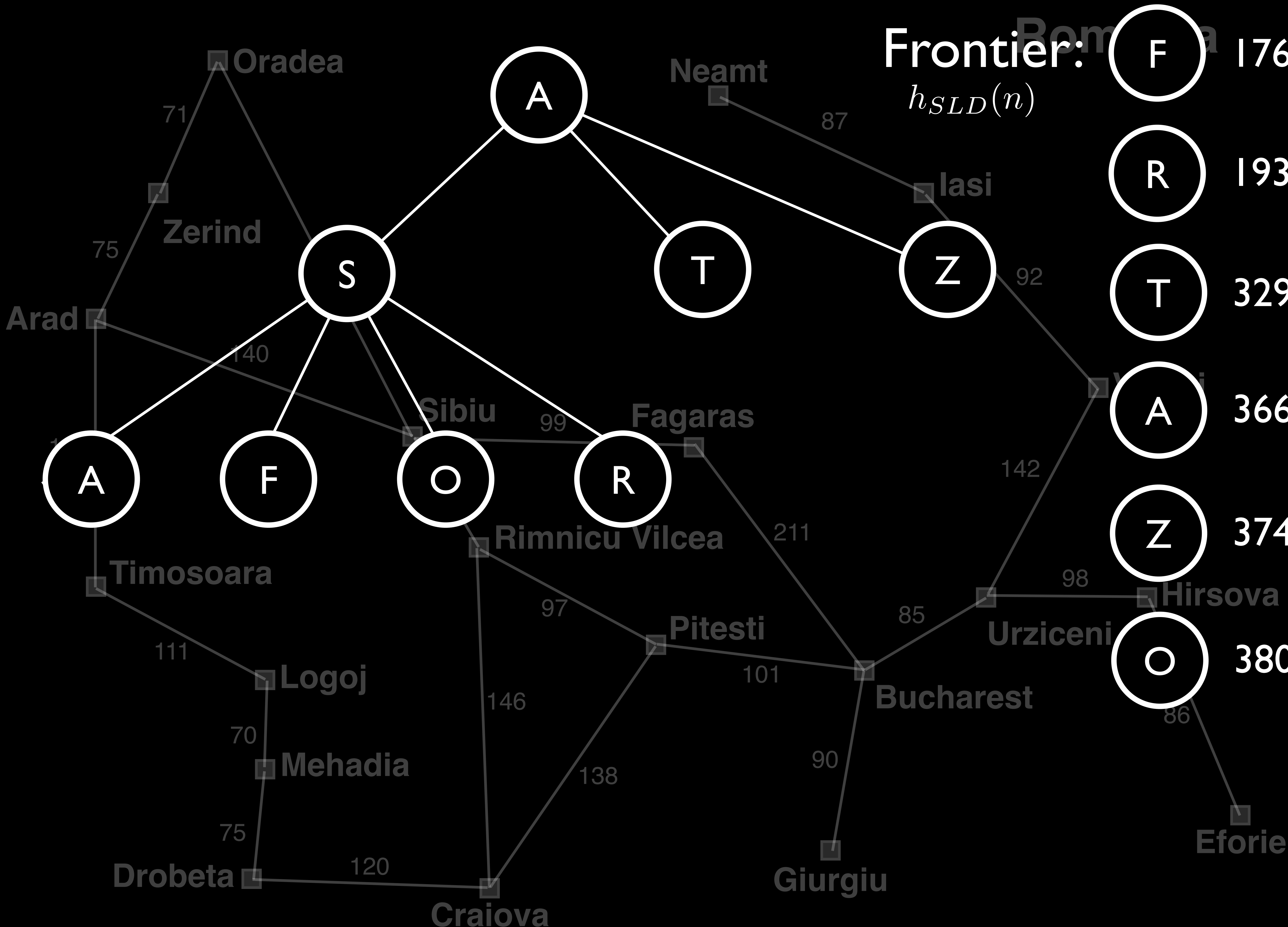

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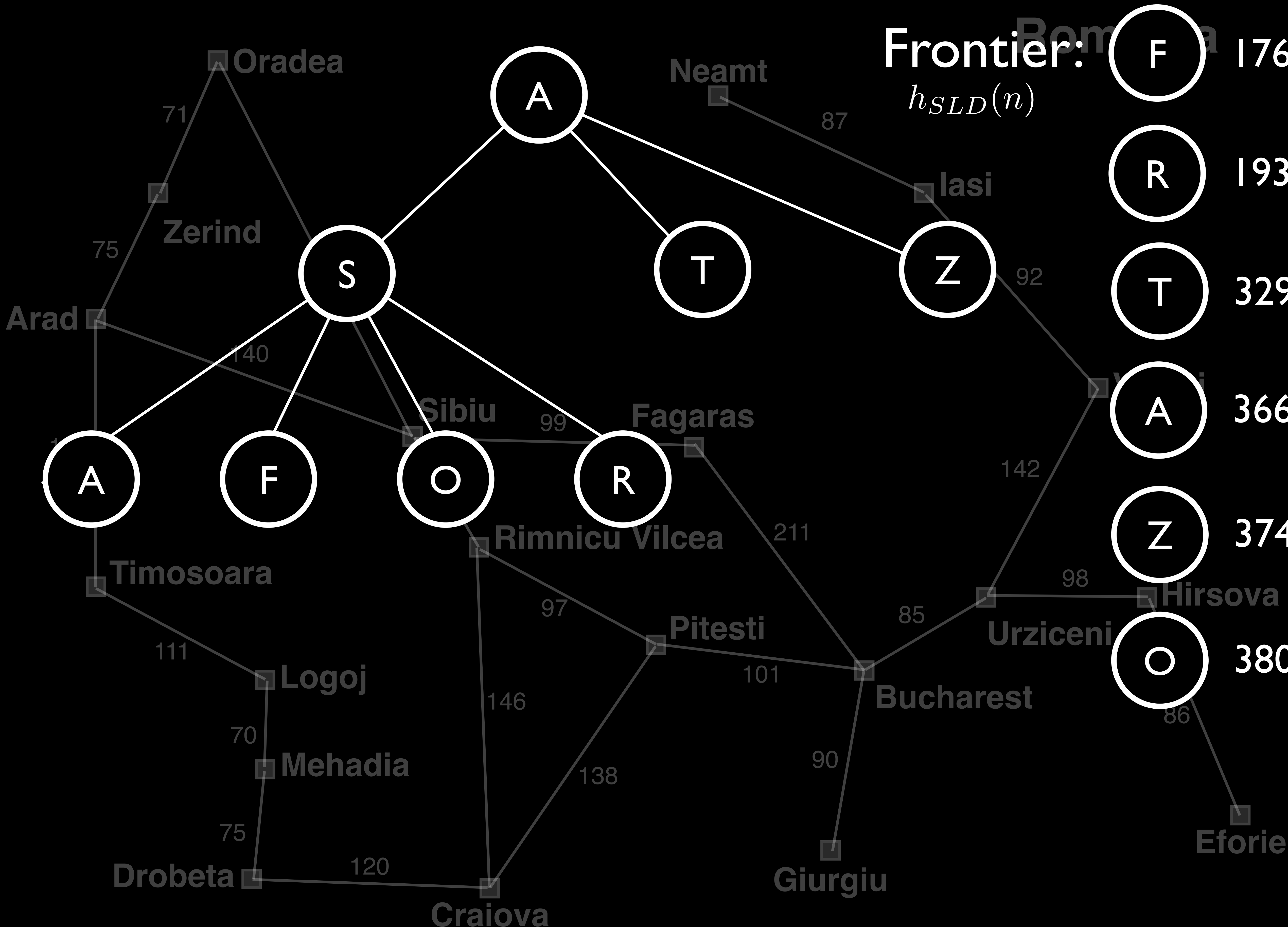

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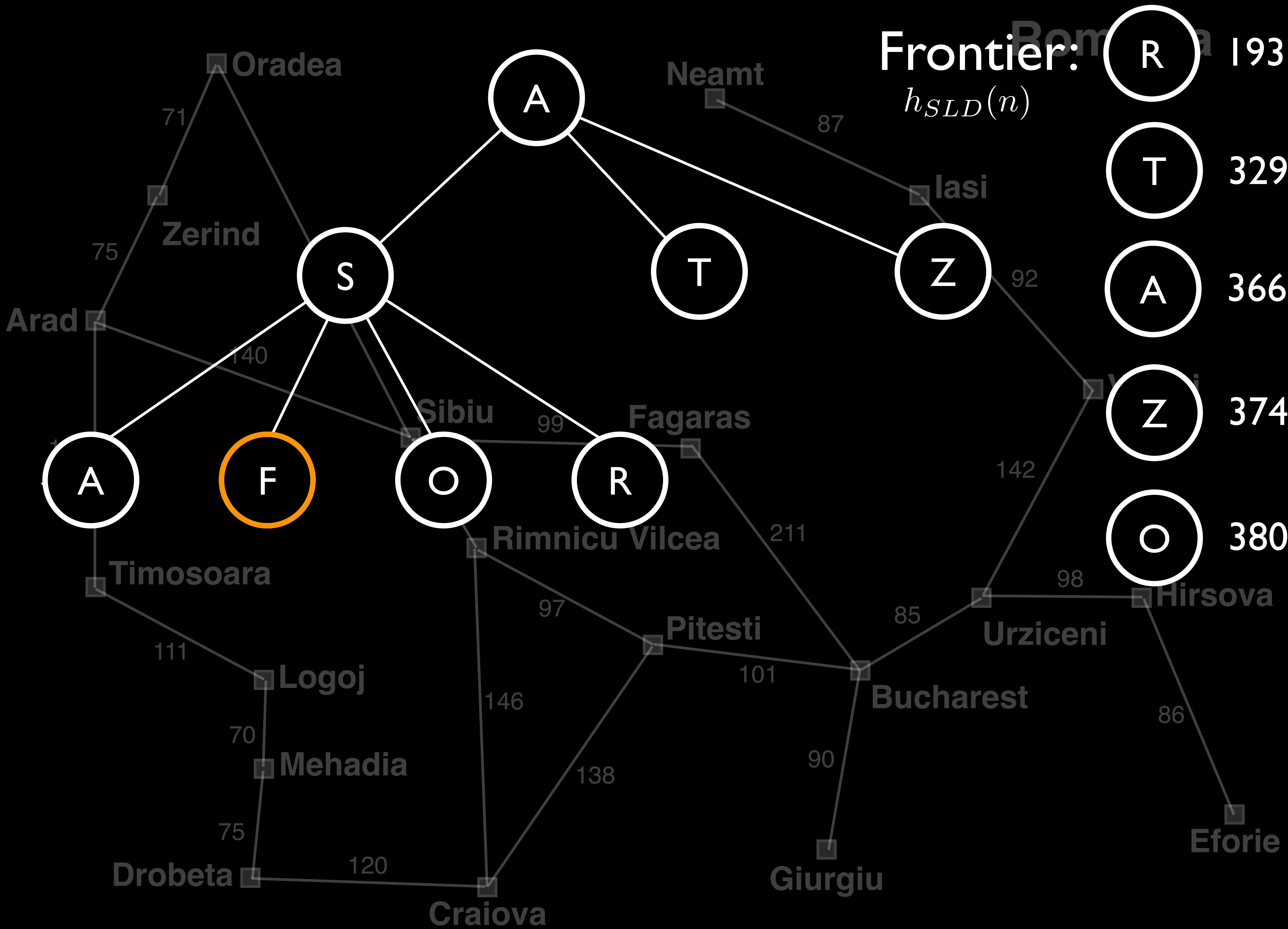

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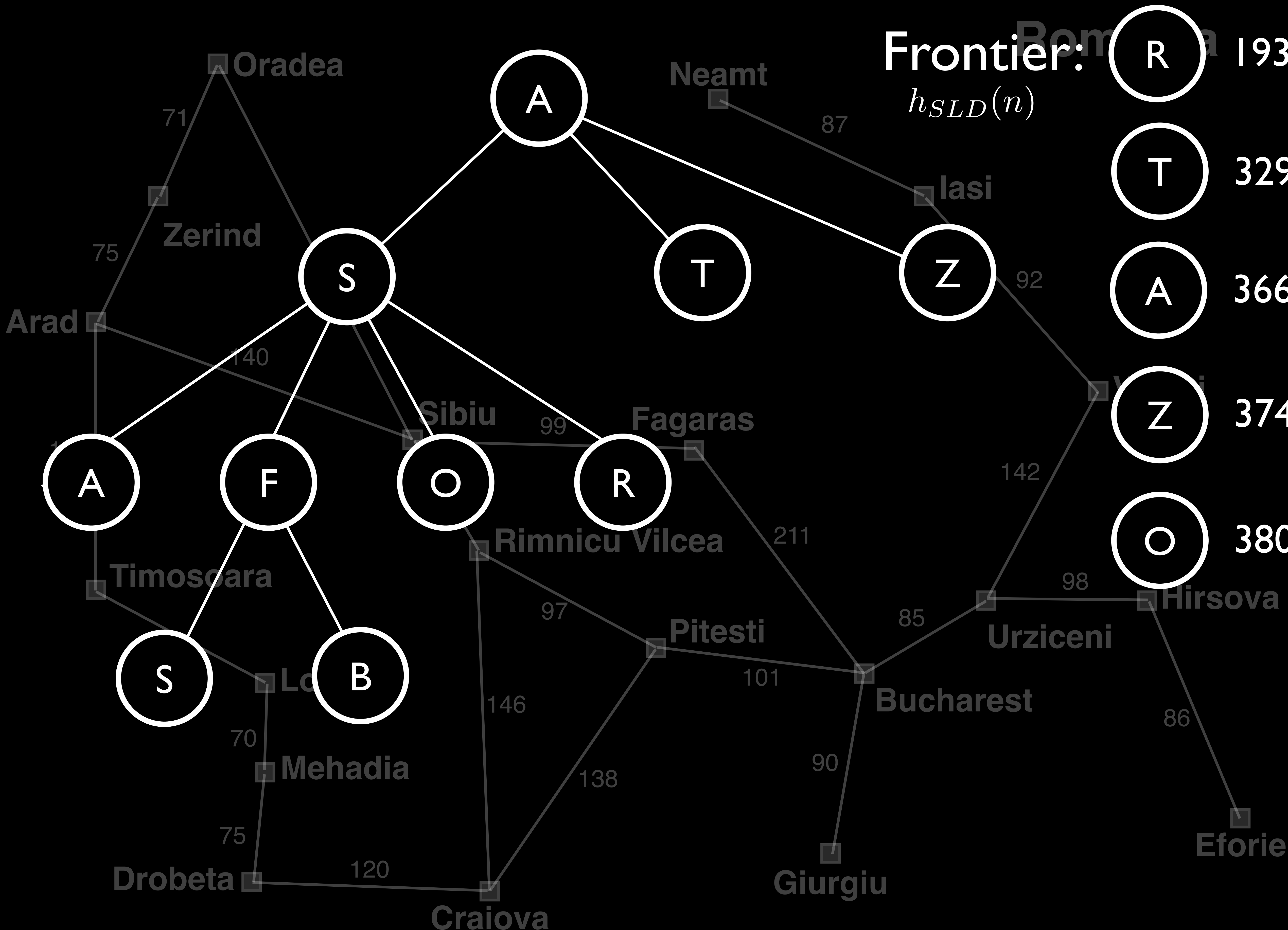

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$h_{SLD}(n)$
R 193
T 329
A 366
Z 374
O 380
A
S
T
Z
A
F
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R
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Oradea
Neamt
71
87
Iasi
Zerind
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Fagaras
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Rimnicu Vilcea
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Timosoara
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Hirsova
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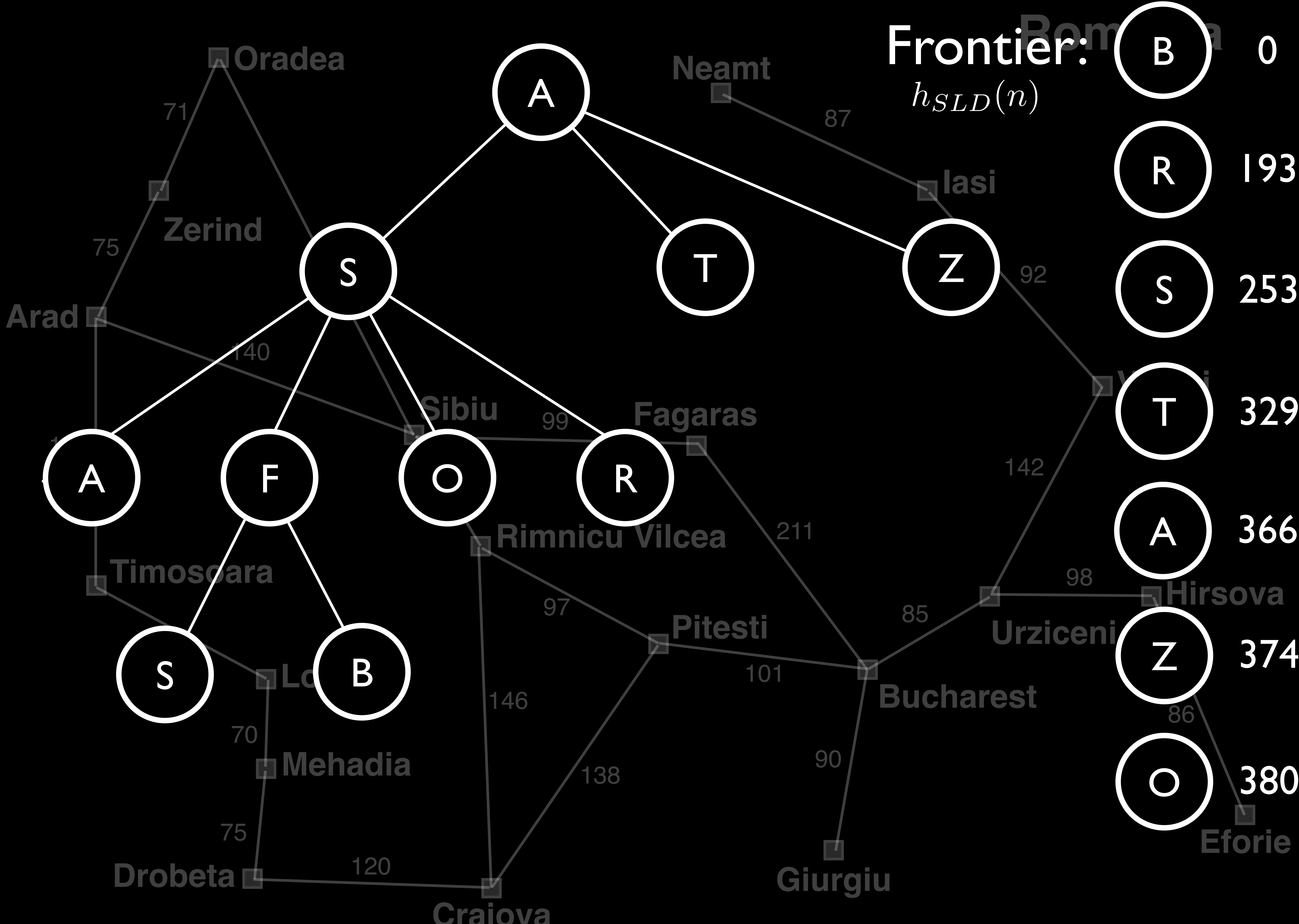

Frontier:
$h_{SLD}(n)$
B 0
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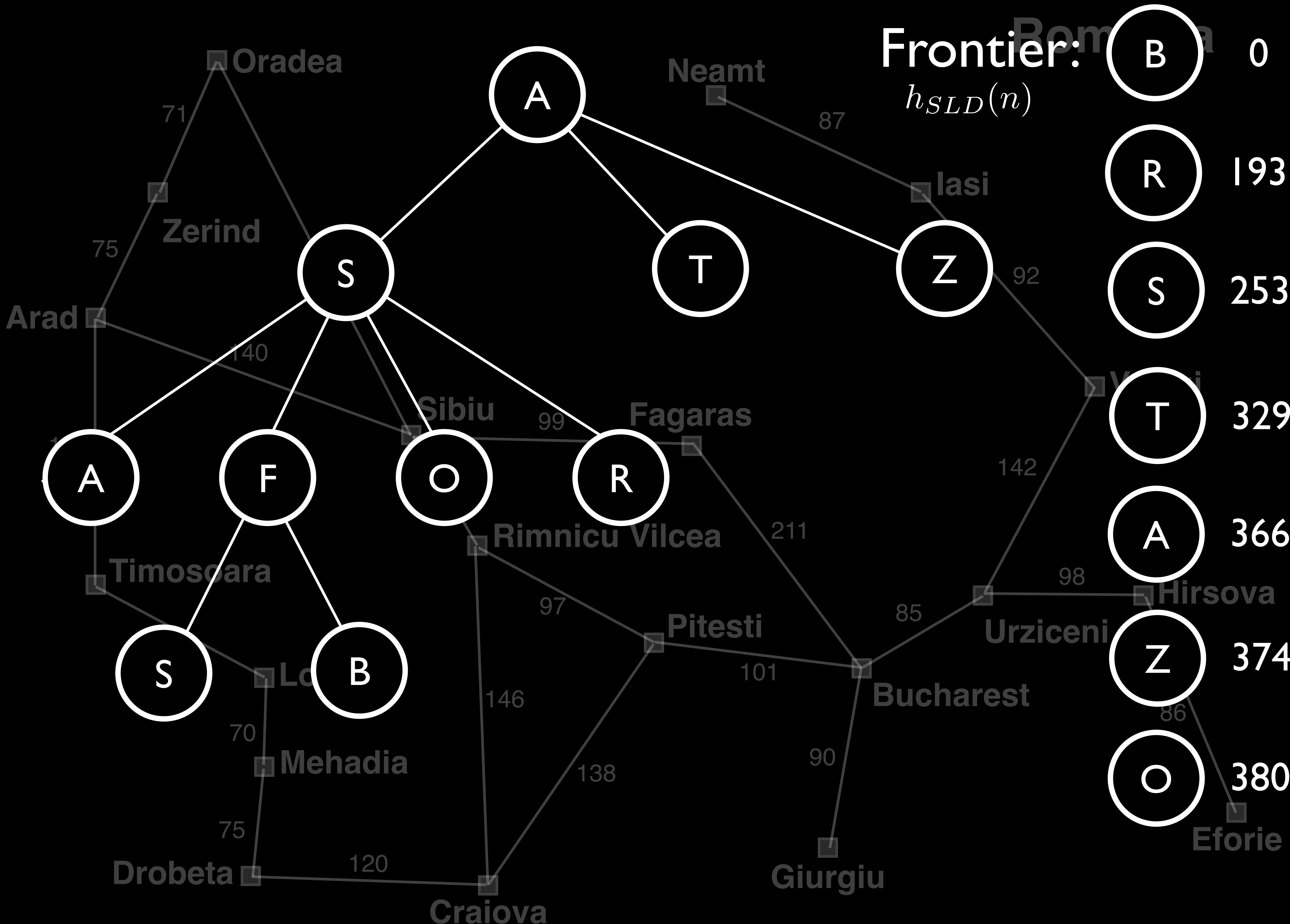
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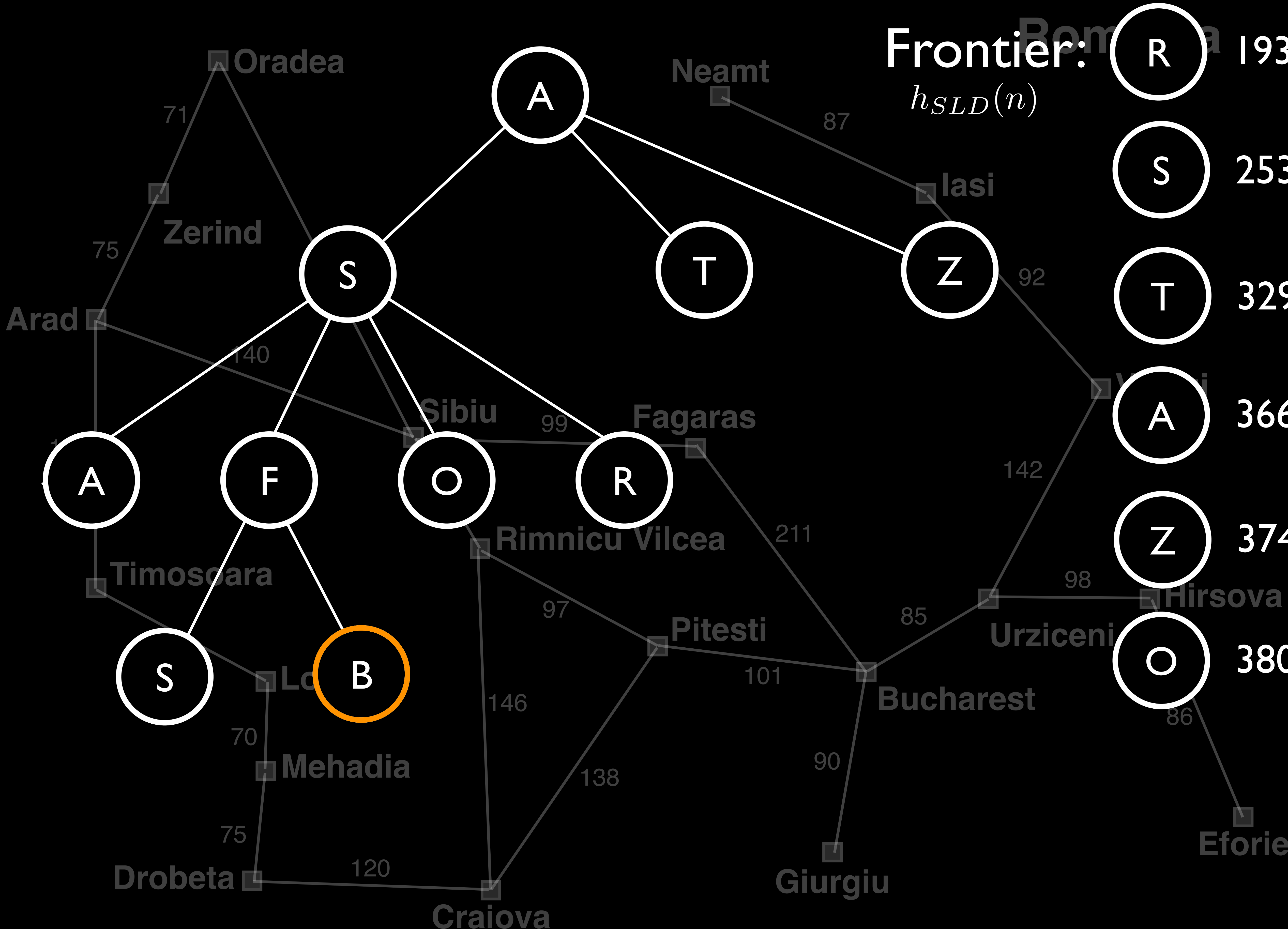
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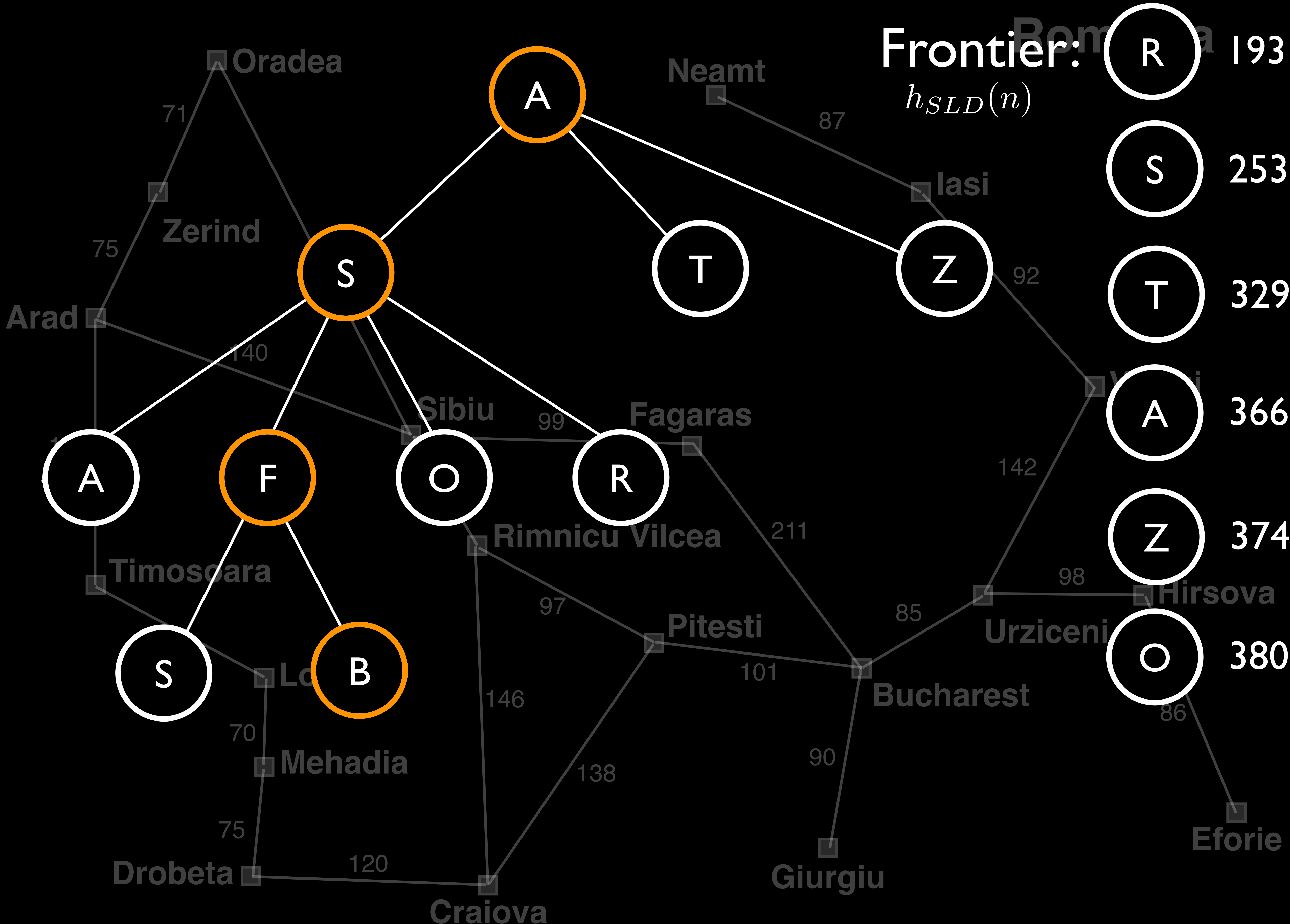

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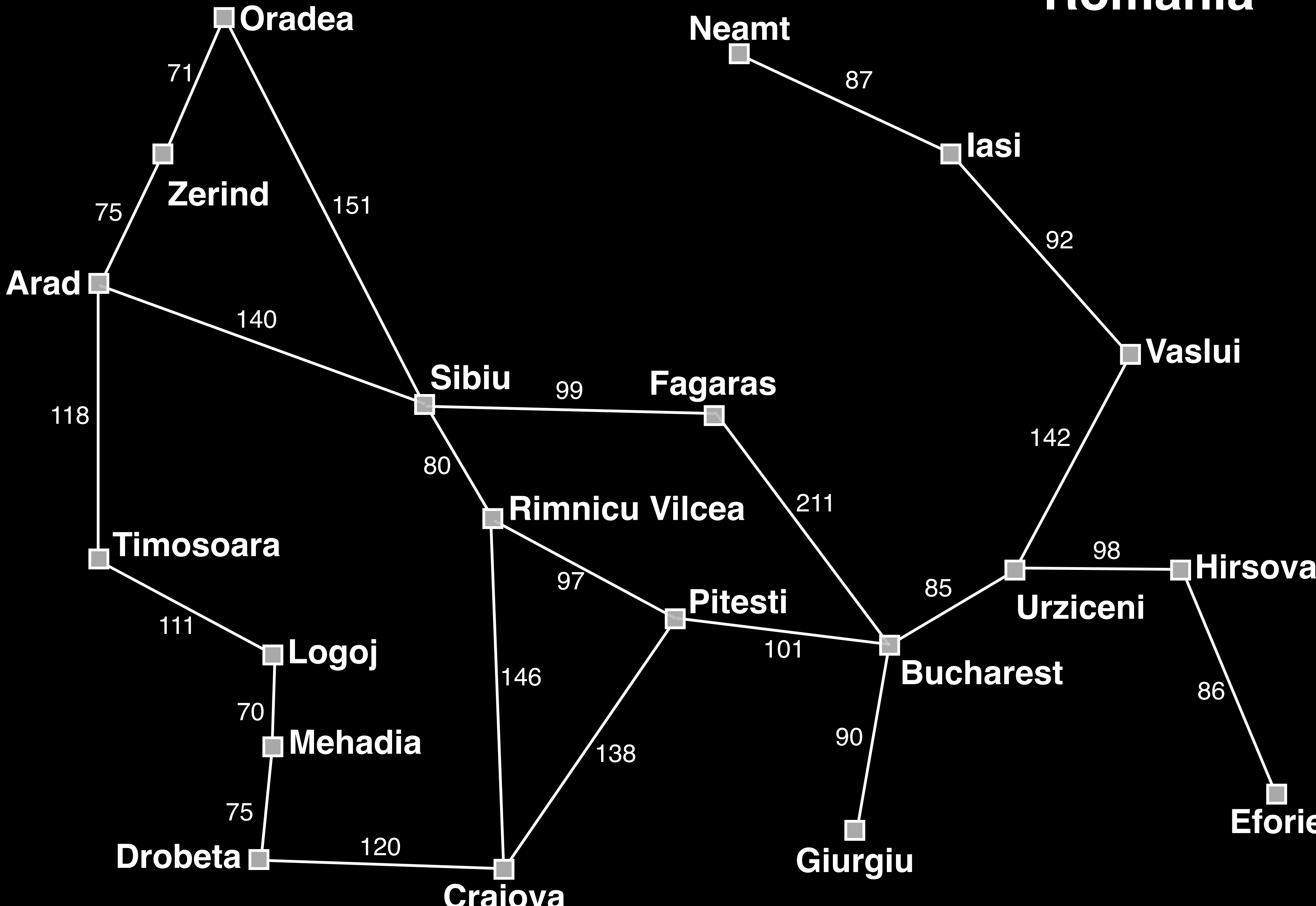
# Romania



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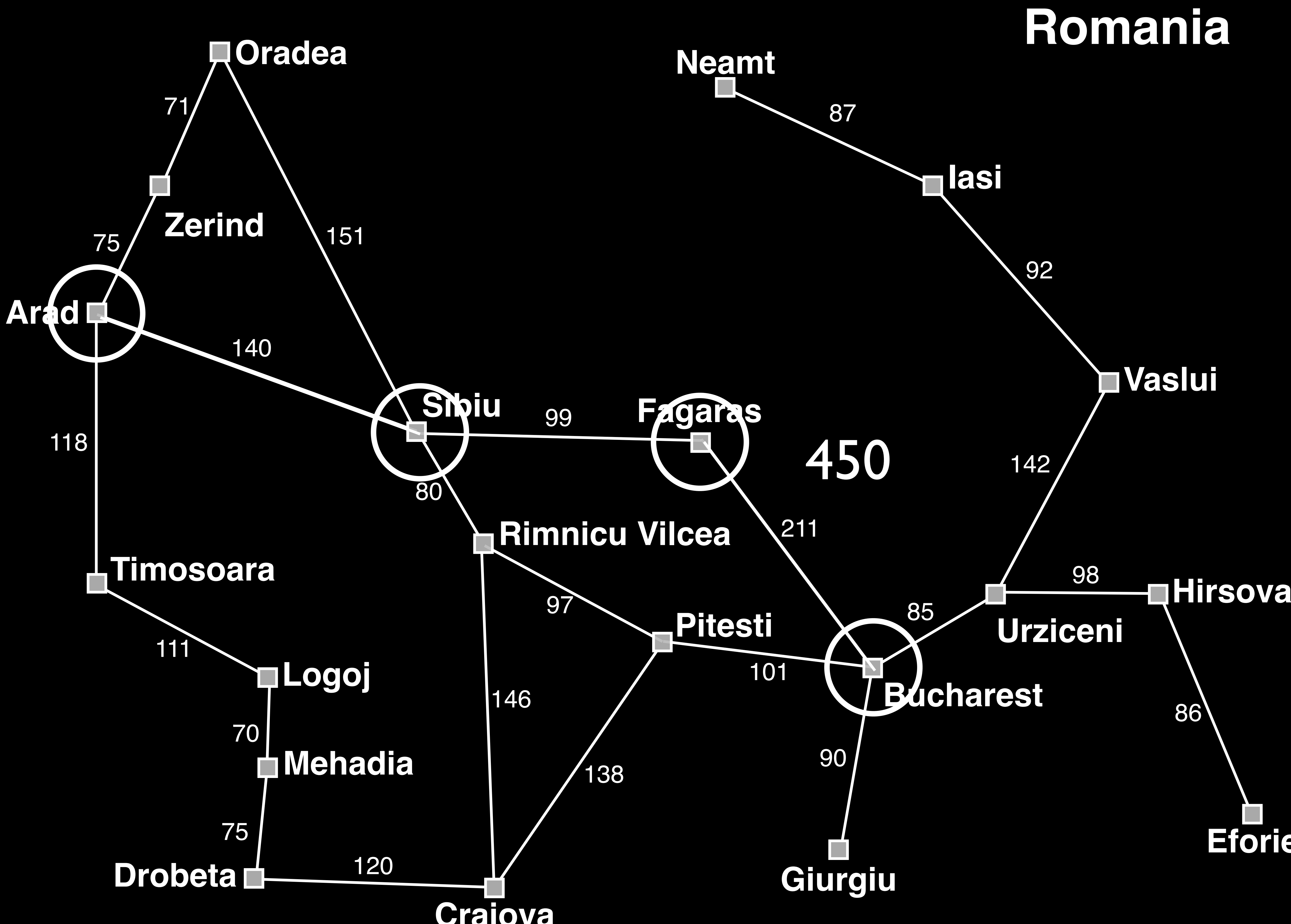

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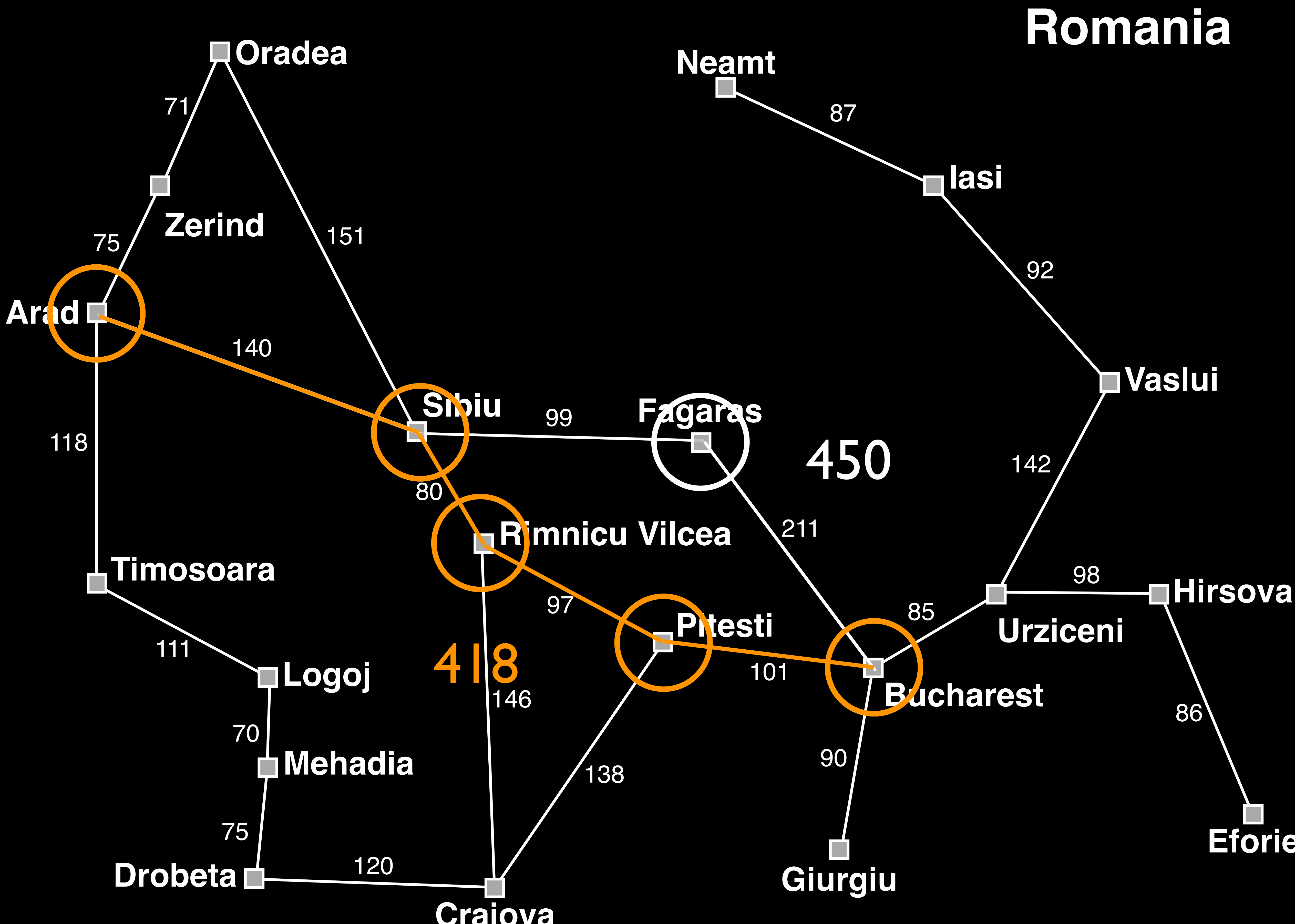

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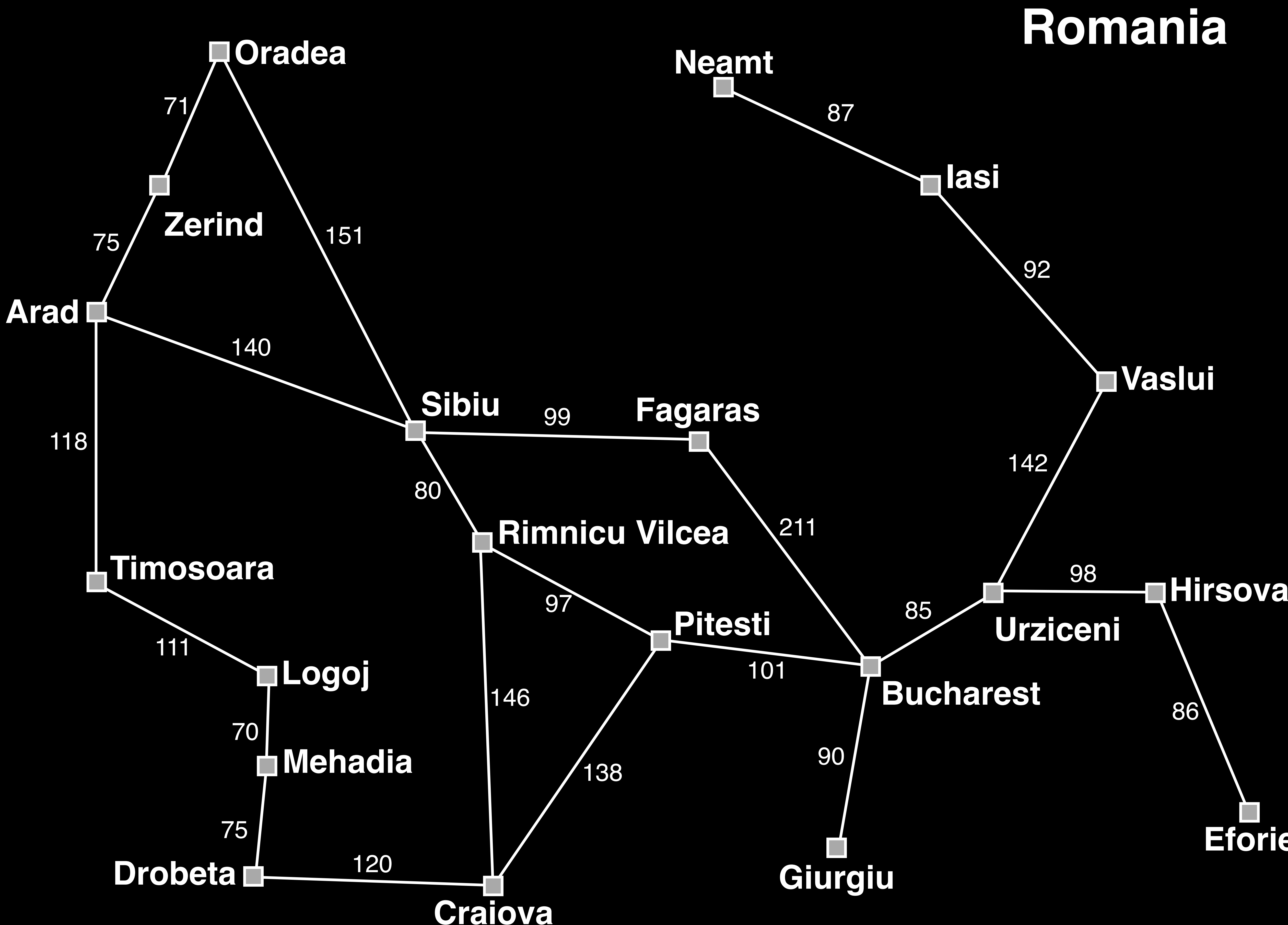
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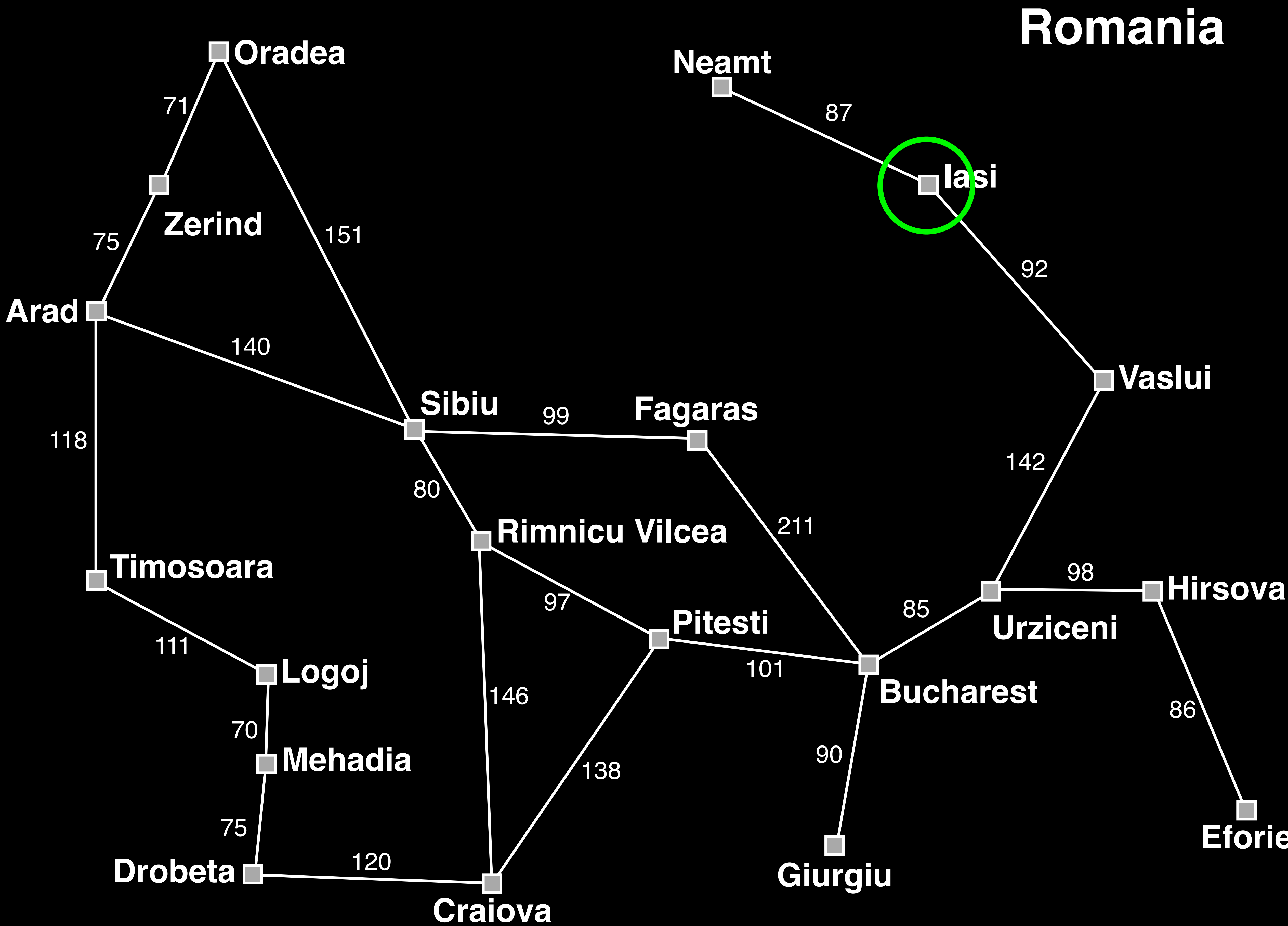
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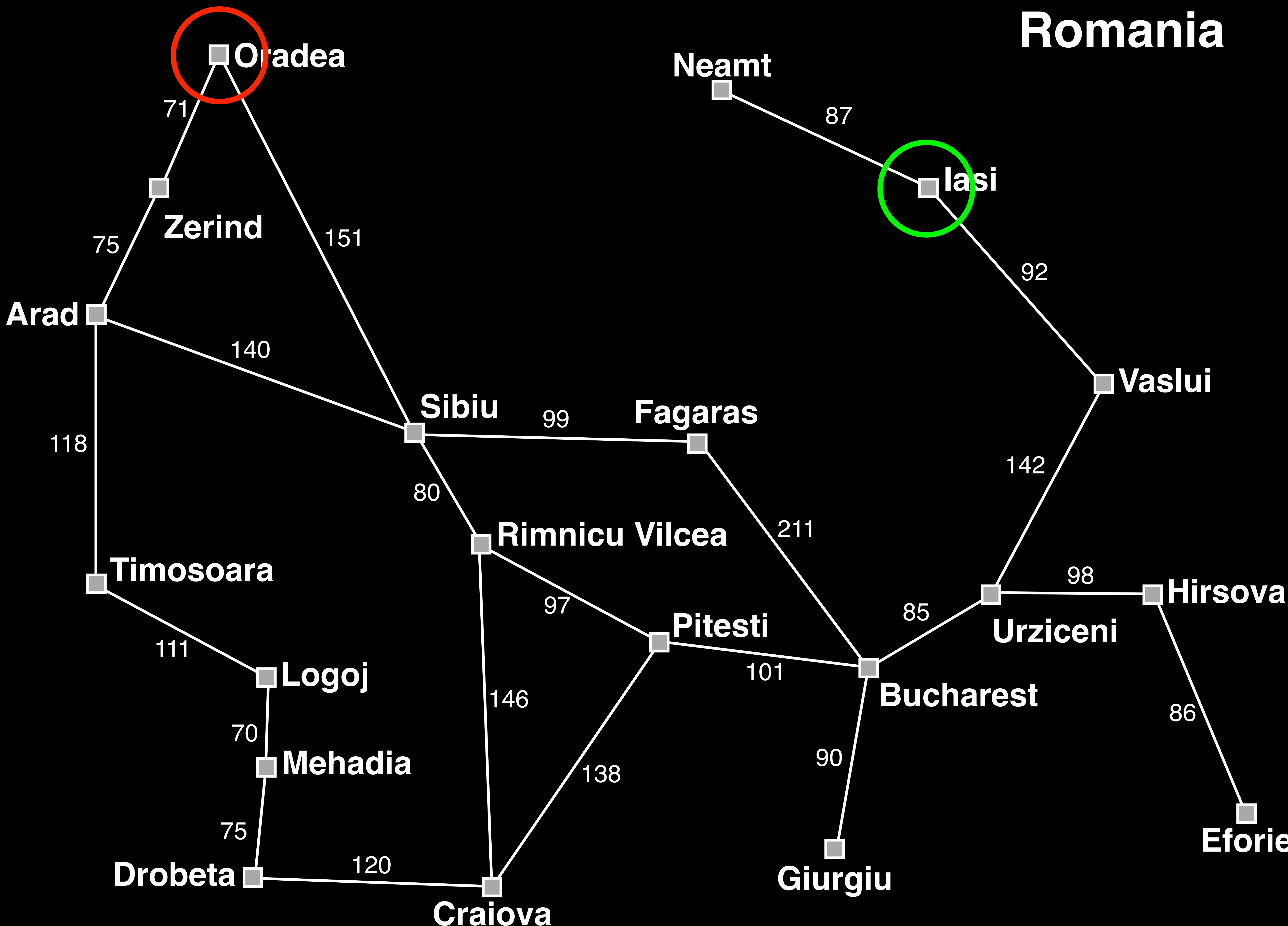

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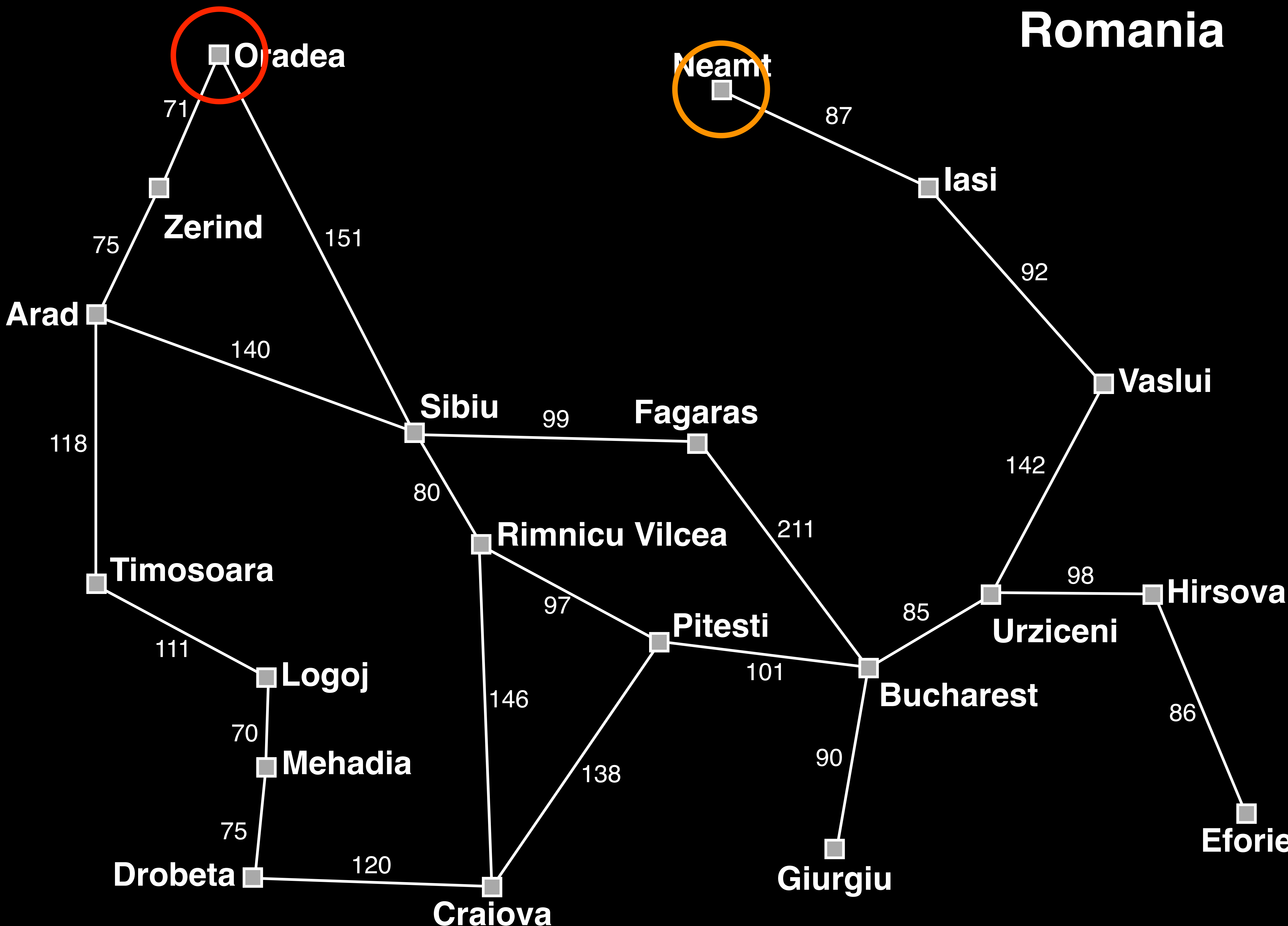
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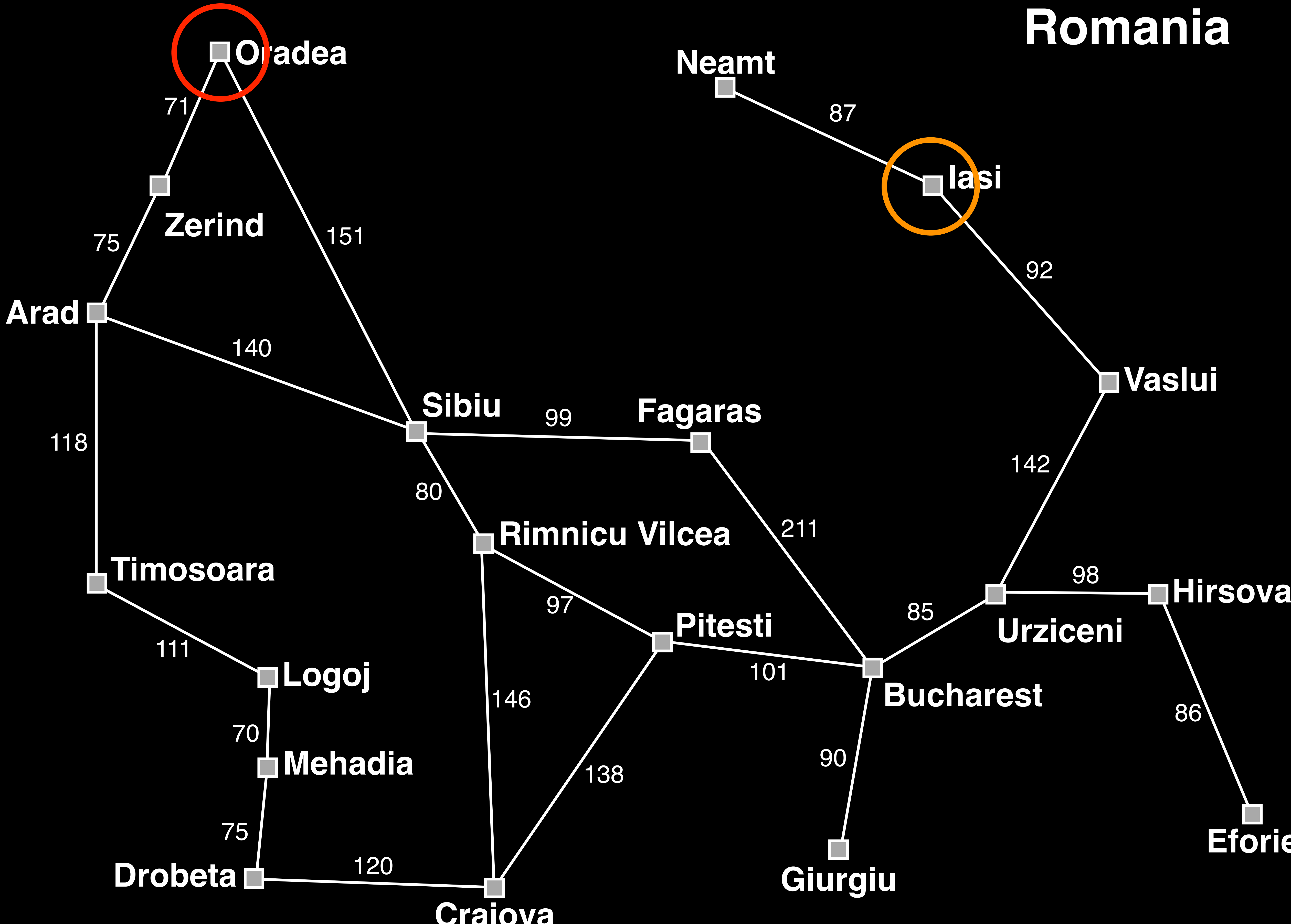
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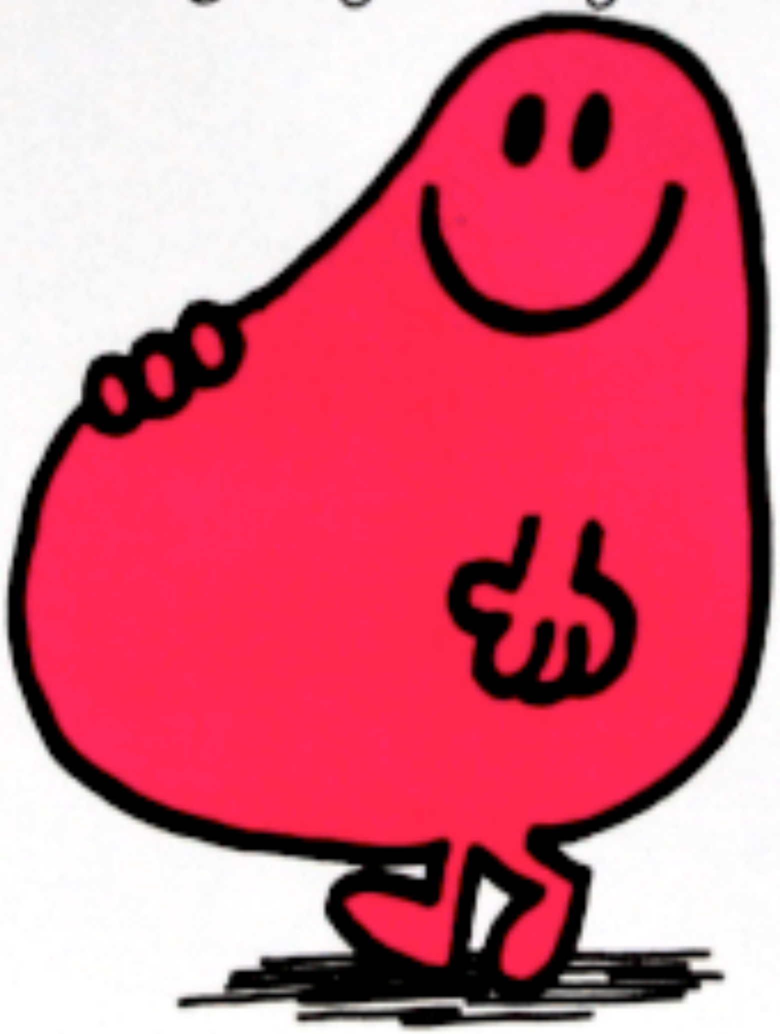
# MR. GREEDY

by Roger Hargreaves



# Greedy Best-First Search

# Greedy Best-First Search

Completeness

# Greedy Best-First Search

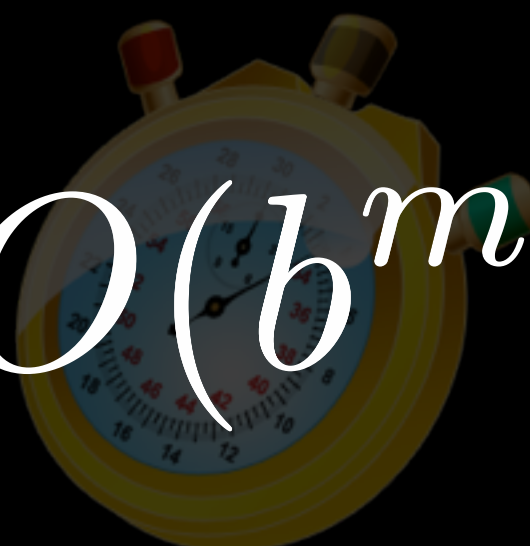

Completeness


Optimality

# Greedy Best-First Search

Completeness

Optimality



$$O(b^m)$$

Time Complexity

# Greedy Best-First Search

Completeness

Optimality

$$O(b^m)$$

Time Complexity

$$O(b^m)$$

Space Complexity

Evaluation function

$$f(n)$$

Evaluation function

$$f(n) = h(n)$$

Estimated cost of cheapest path from *n* to a goal node

Evaluation function

$$f(n) = g(n) + h(n)$$

Known cost of path from start node to node *n*

Estimated cost of cheapest path from *n* to a goal node

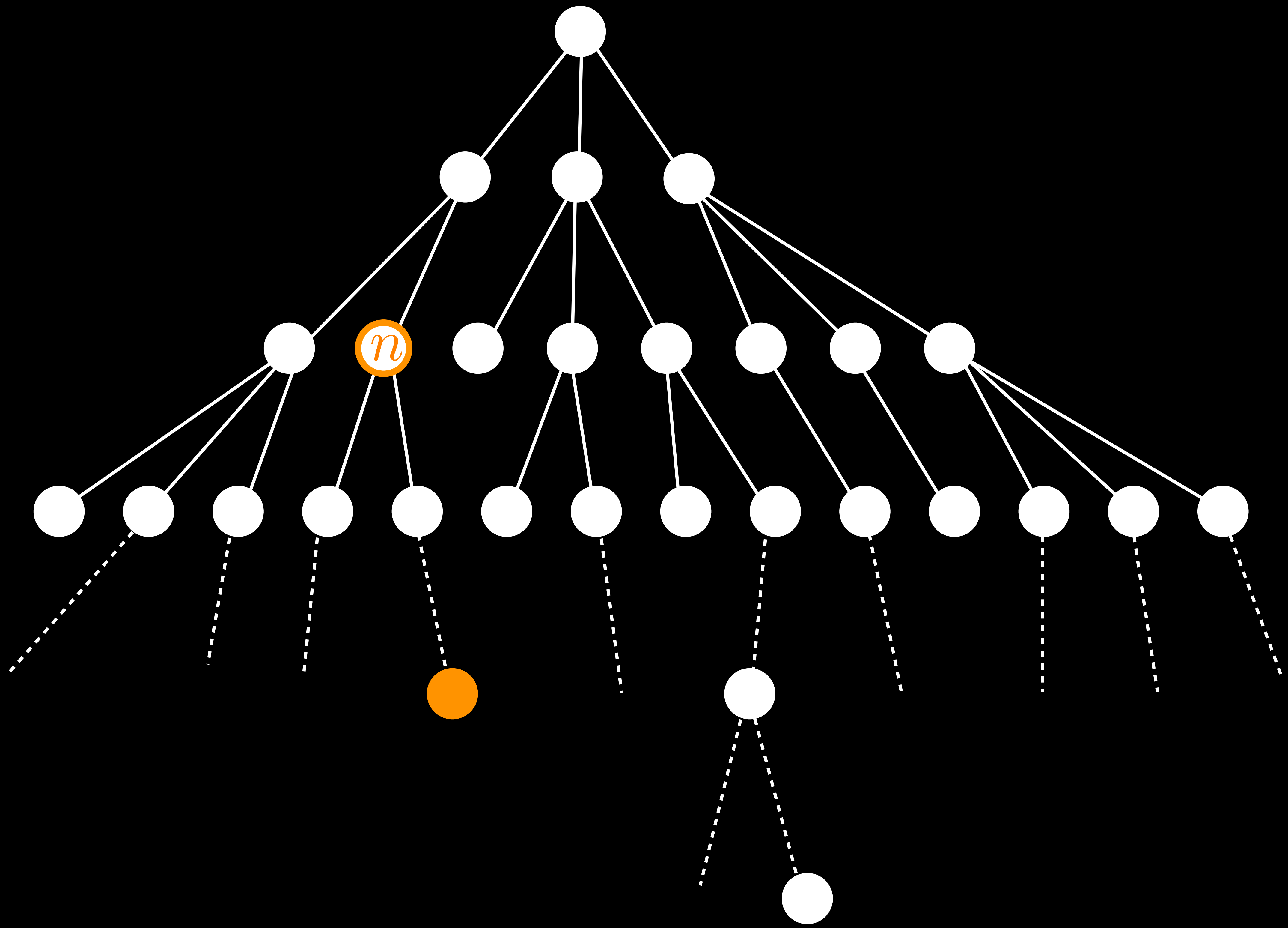
# A*

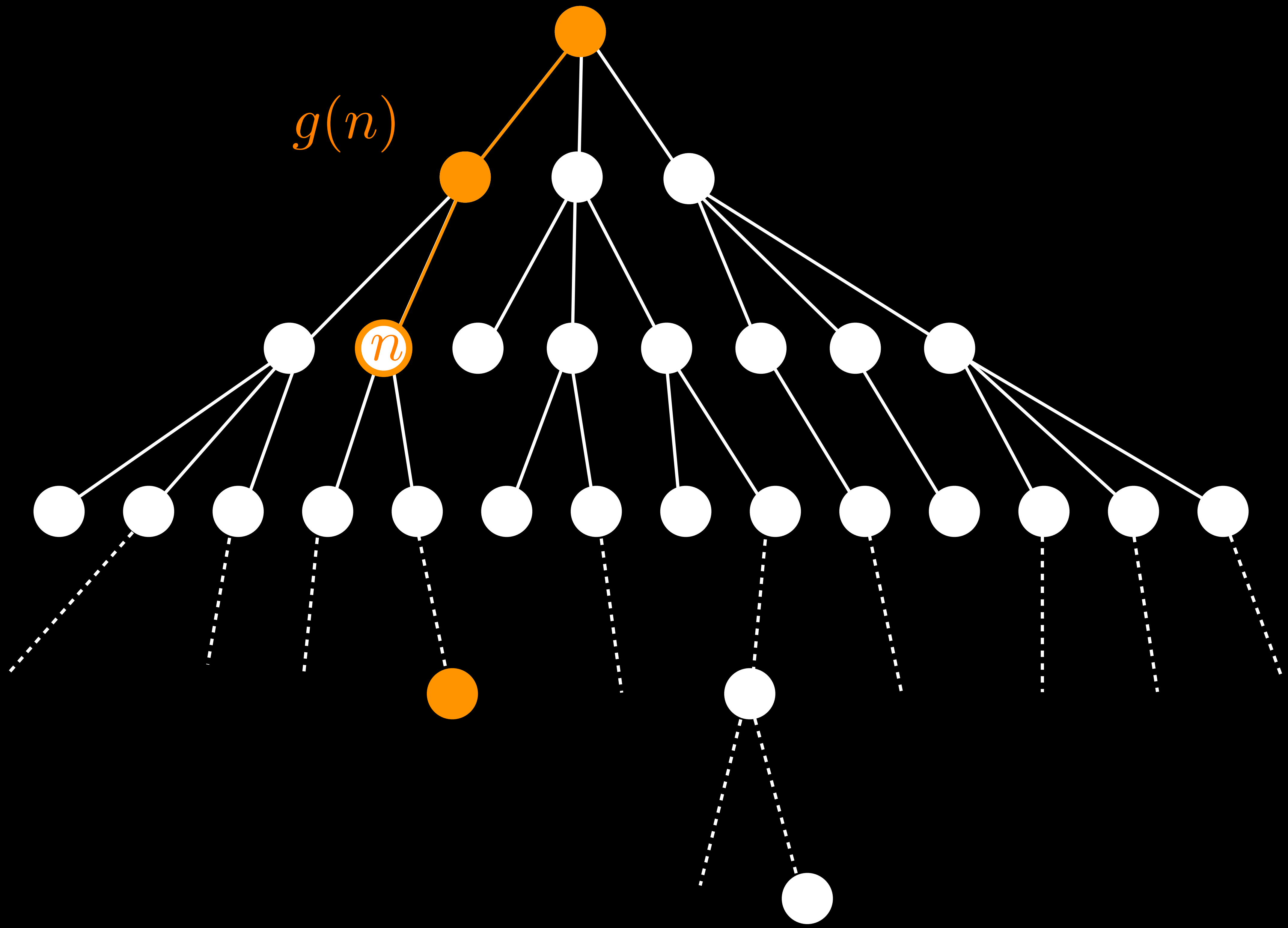
Evaluation function

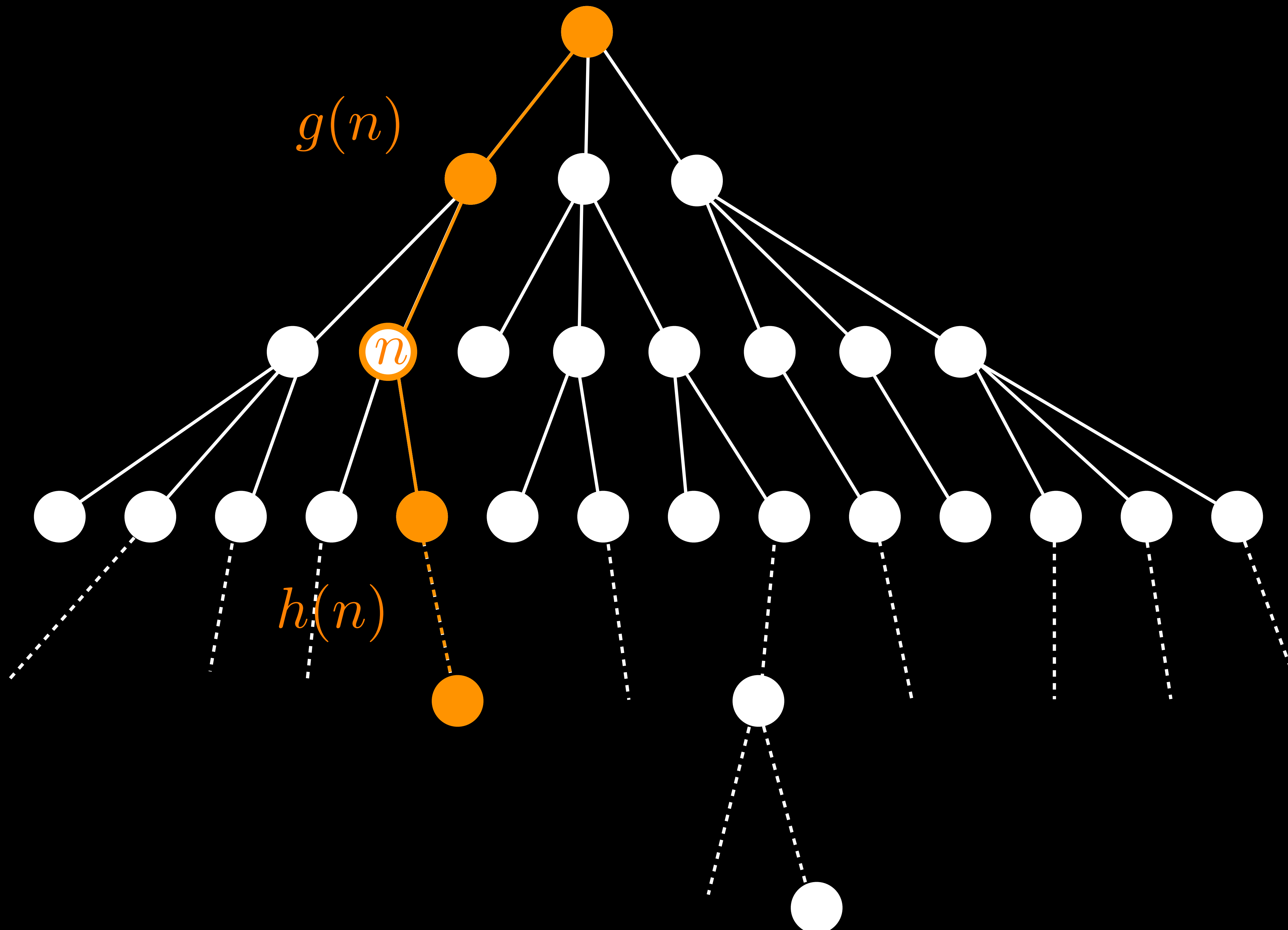
$$f(n) = g(n) + h(n)$$

Known cost of path from start node to node *n*

Estimated cost of cheapest path from *n* to a goal node


n


g(n)
n


$g(n)$
$n$
$h(n)$

Evaluation function

$$f(n) = g(n) + h(n)$$

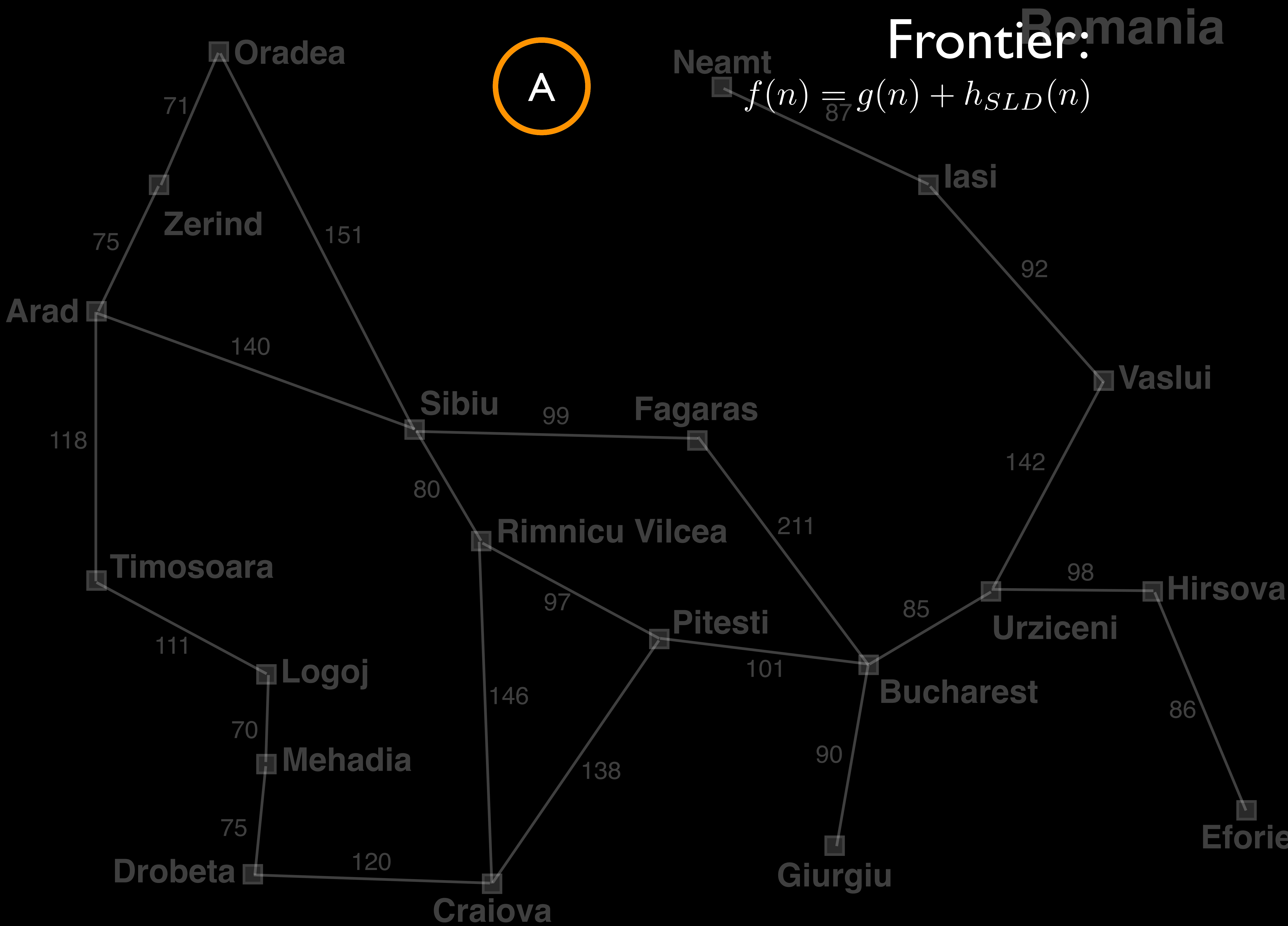
$=$ Estimated cost of cheapest solution through *n*

Frontier: A 0+366 =366

$$f(n) = g(n) + h_{SLD}(n)$$

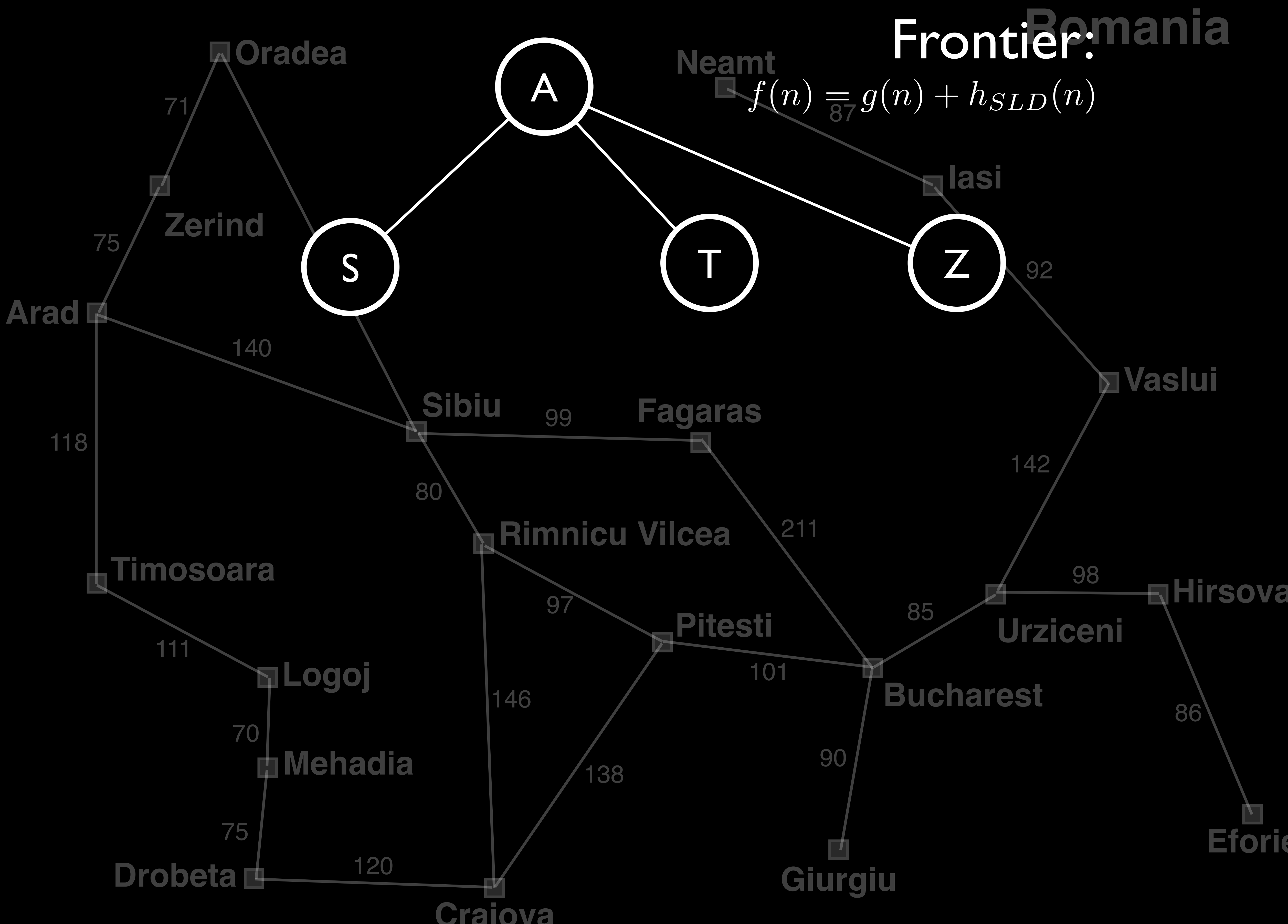
## Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$



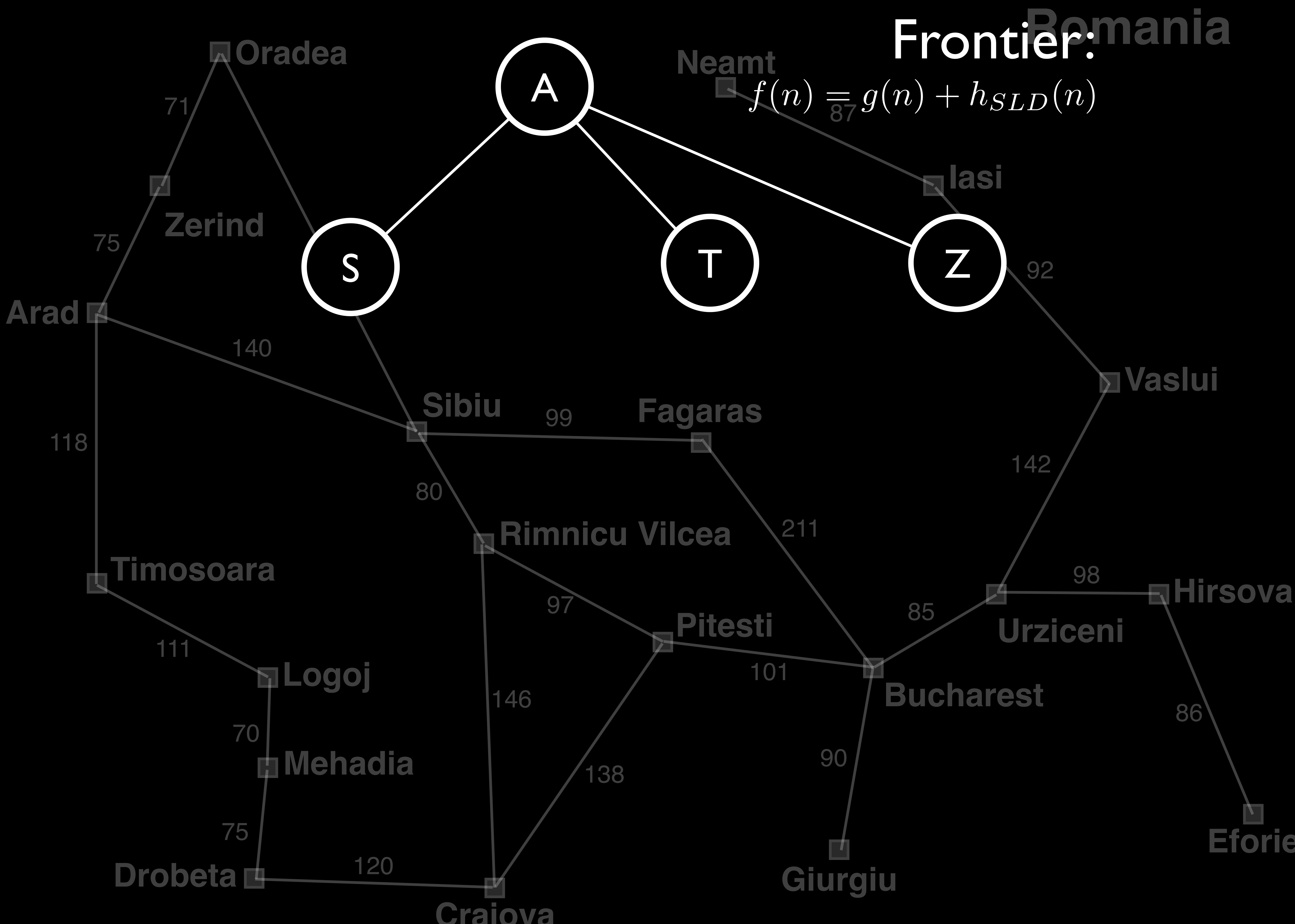
Frontier:

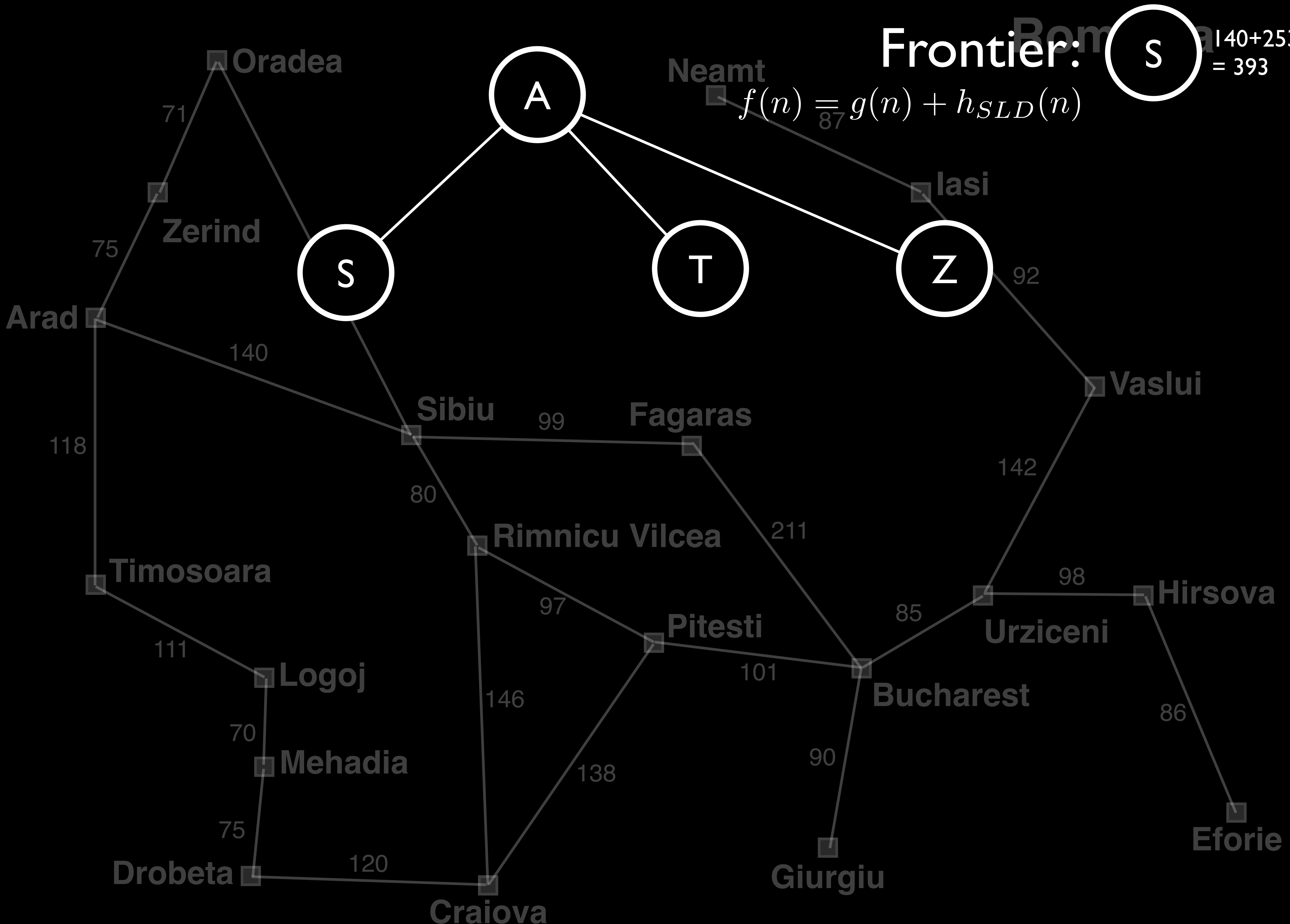
$$f(n) = g(n) + h_{SLD}(n)$$



# Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$




Frontier:
S
140+253
= 393
$f(n) = g(n) + h_{SLD}(n)$
Romania
A
S
T
Z
Oradea
Neamt
71
87
Iasi
Zerind
75
92
Arad
140
Vaslui
Sibiu
99
Fagaras
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142
80
Rimnicu Vilcea
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Timosoara
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Hirsova
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Pitesti
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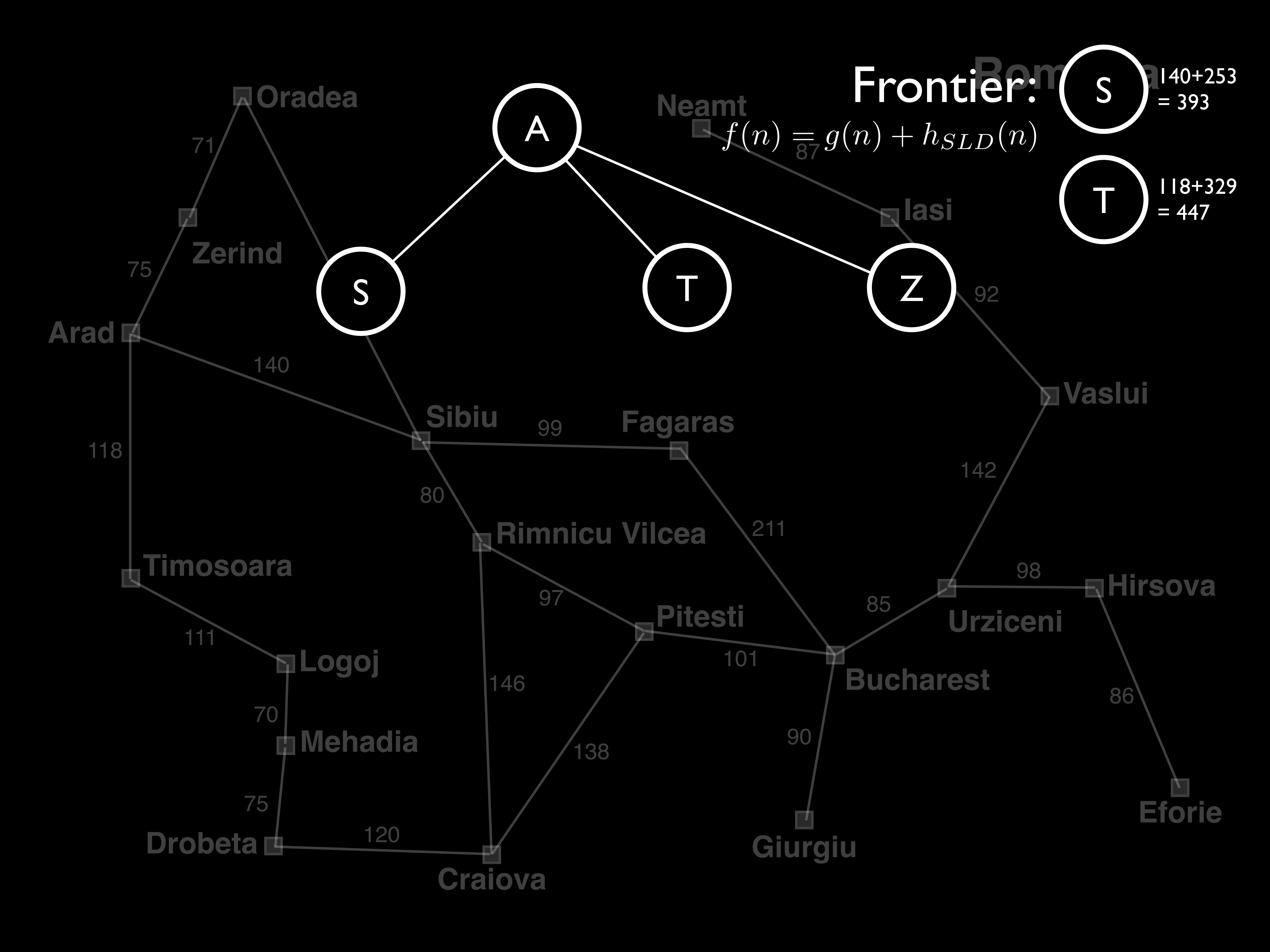

Frontier:
$f(n) = g(n) + h_{SLD}(n)$
S
140+253
= 393
T
118+329
= 447
A
S
T
Z
Oradea
Neamt
Iasi
Zerind
Arad
Sibiu
Fagaras
Vaslui
Rimnicu Vilcea
Timosoara
Urziceni
Hirsova
Pitesti
Logoj
Bucharest
Mehadia
Drobeta
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Giurgiu
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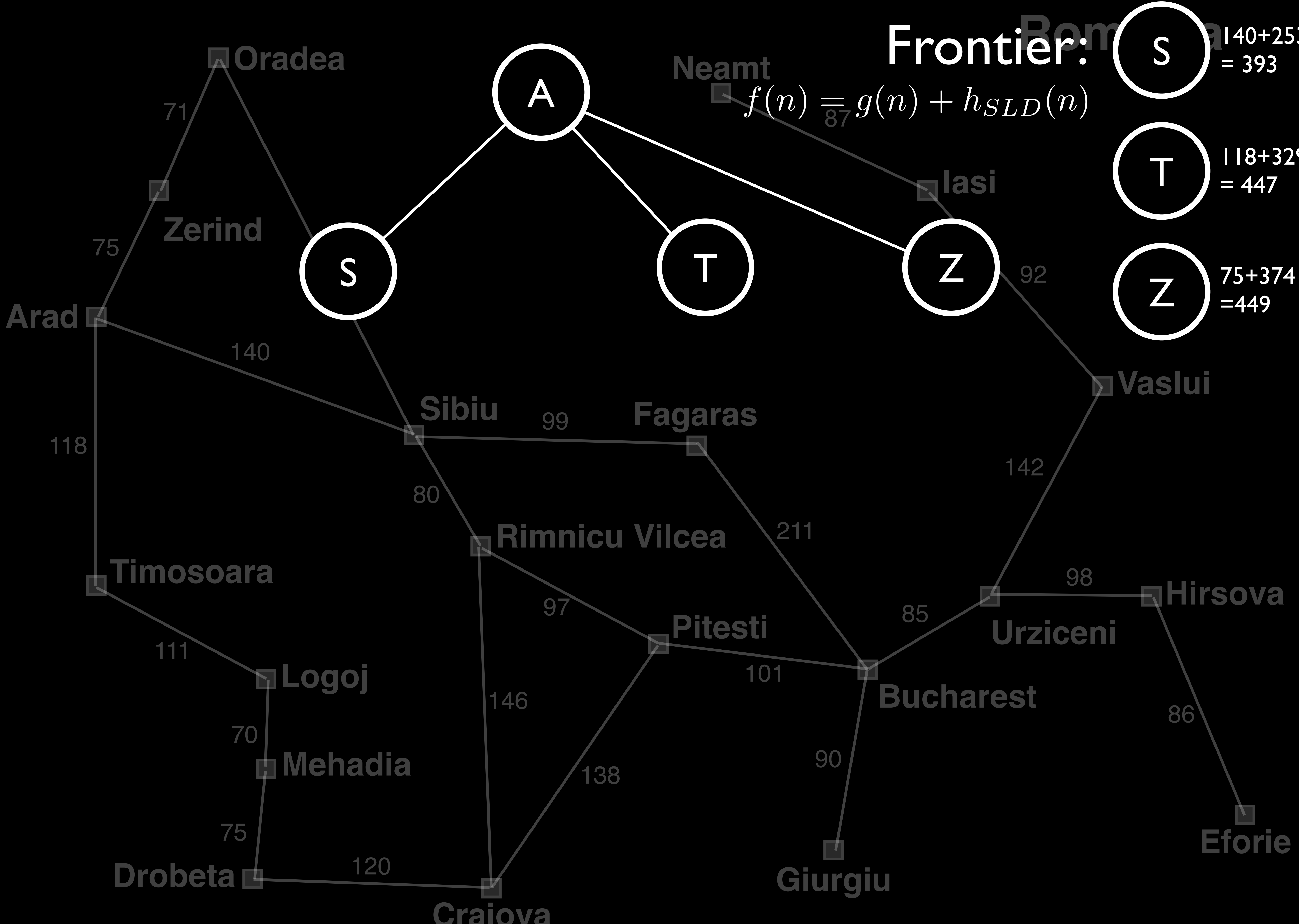
Frontier:
$f(n) = g(n) + h_{SLD}(n)$
S 140+253 = 393
T 118+329 = 447
Z 75+374 =449
A
S
T
Z
Romania
Oradea
Neamt
Zerind
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Arad
Sibiu
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Rimnicu Vilcea
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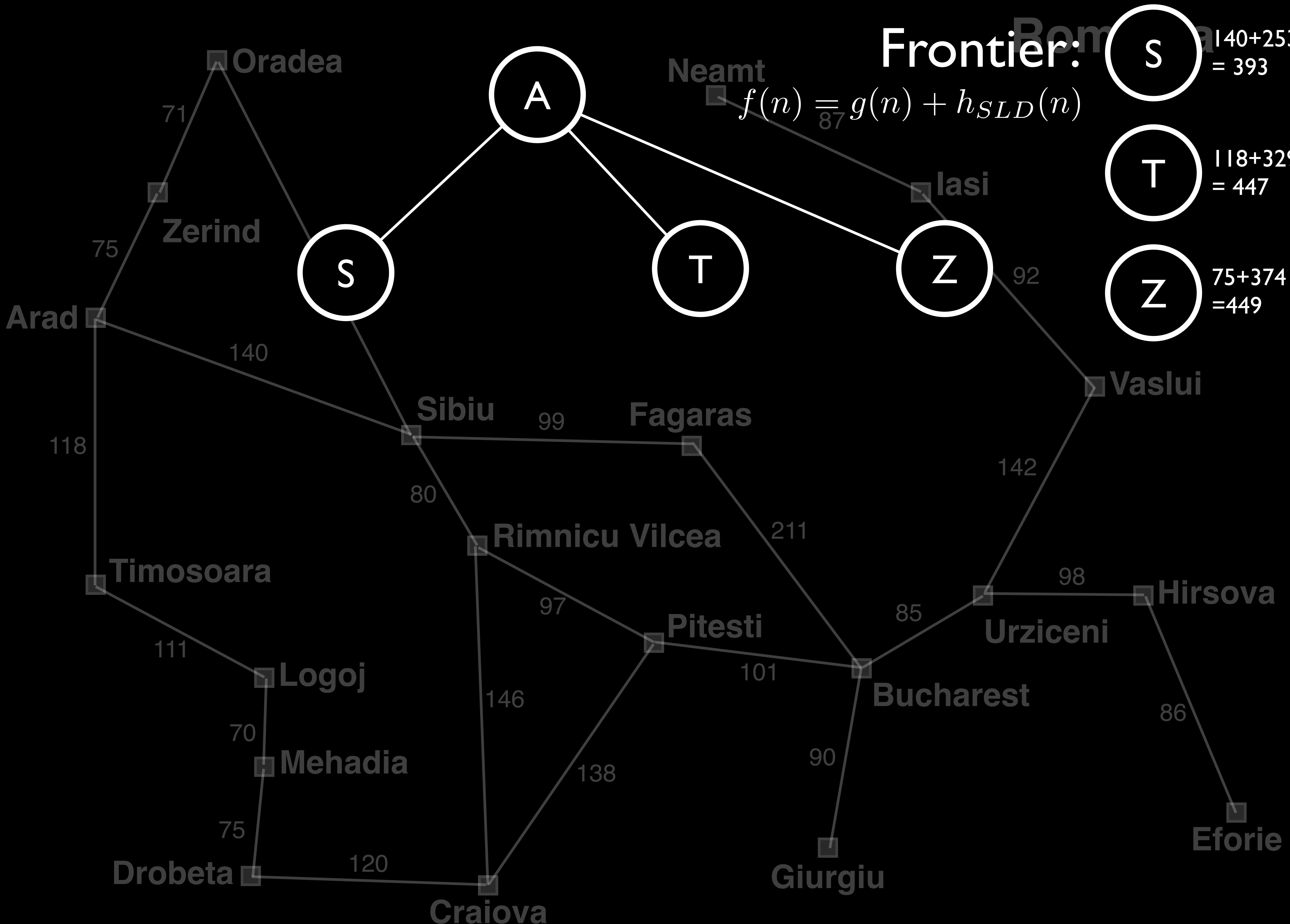
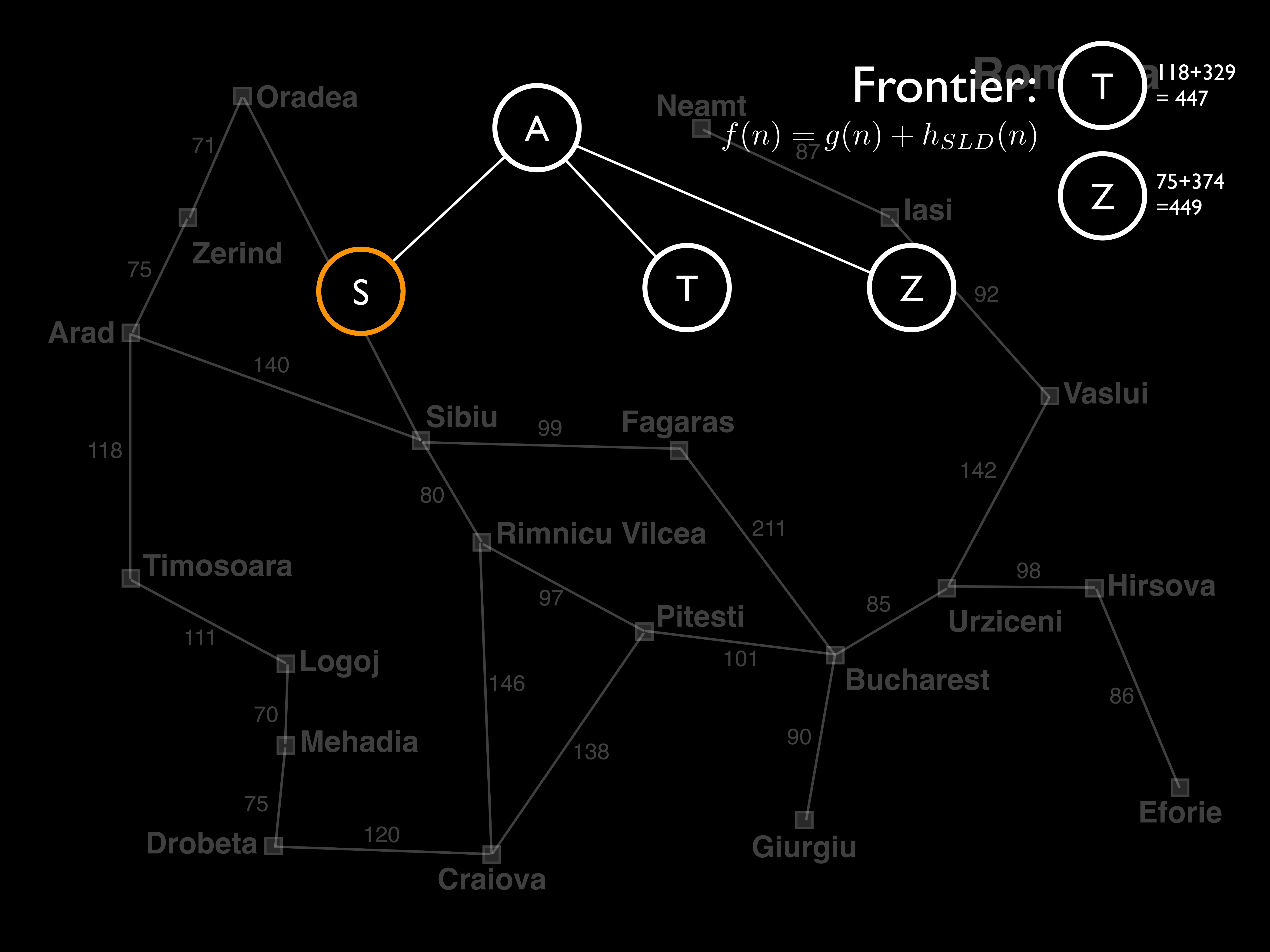
## Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$





Frontier:
$f(n) = g(n) + h_{SLD}(n)$
T
118+329
= 447
Z
75+374
=449
A
S
T
Z
Oradea
Neamt
Zerind
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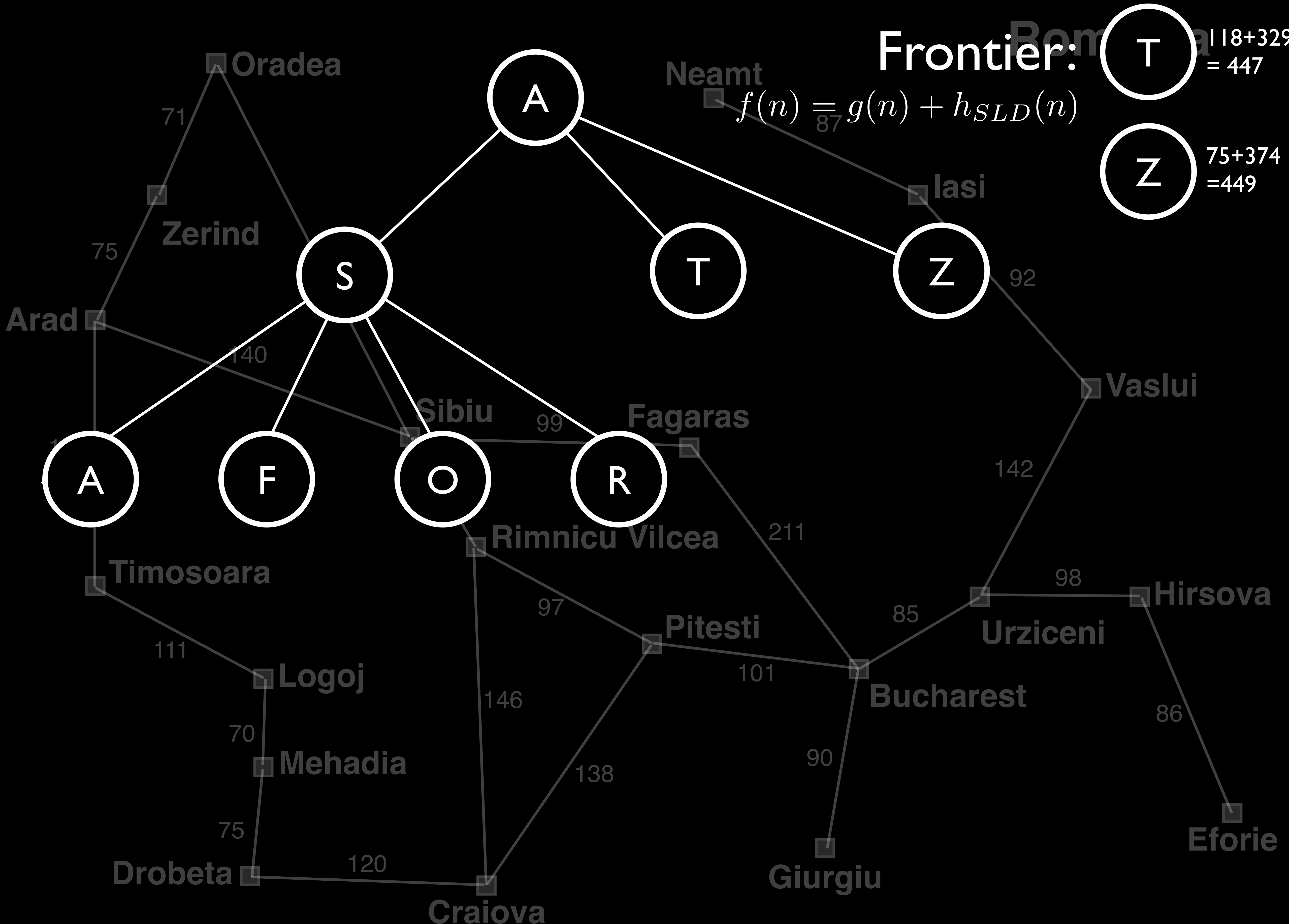
Frontier:
T
118+329
= 447
Z
75+374
=449
$f(n) = g(n) + h_{SLD}(n)$
A
S
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F
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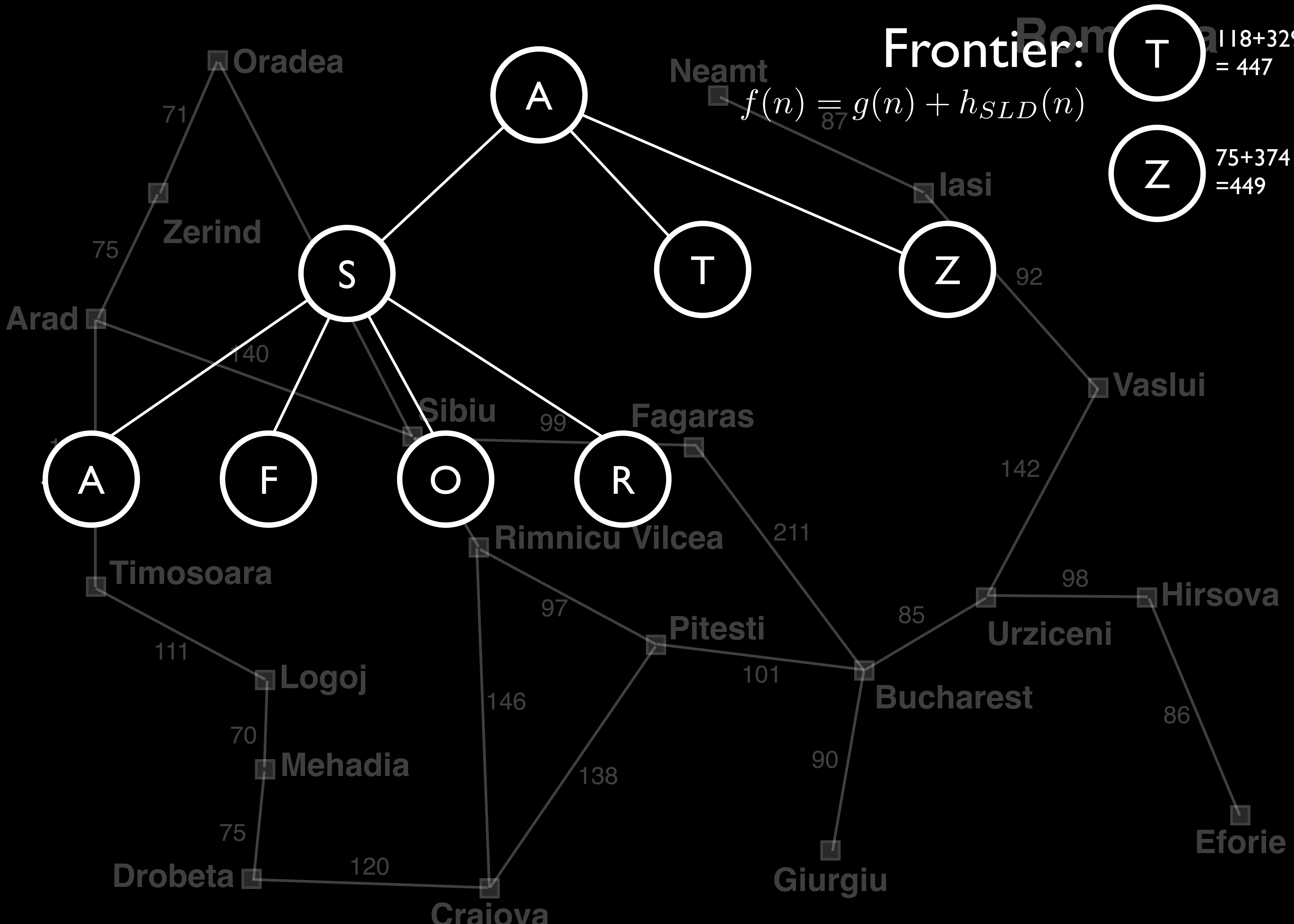
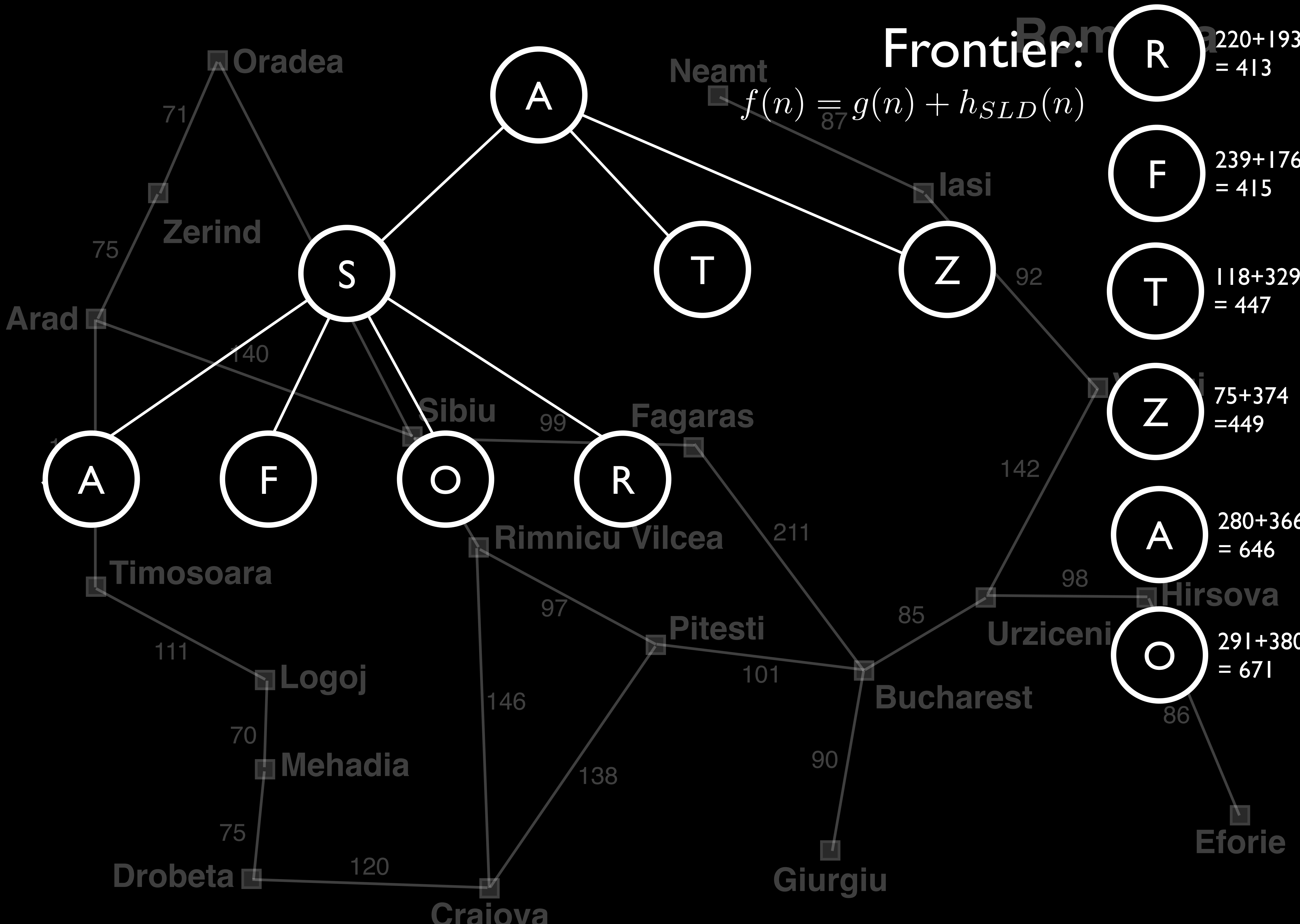
Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$





Frontier:
$f(n) = g(n) + h_{SLD}(n)$
R 220+193 = 413
F 239+176 = 415
T 118+329 = 447
Z 75+374 =449
A 280+366 = 646
O 291+380 = 671
A
S
T
Z
A
F
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Oradea
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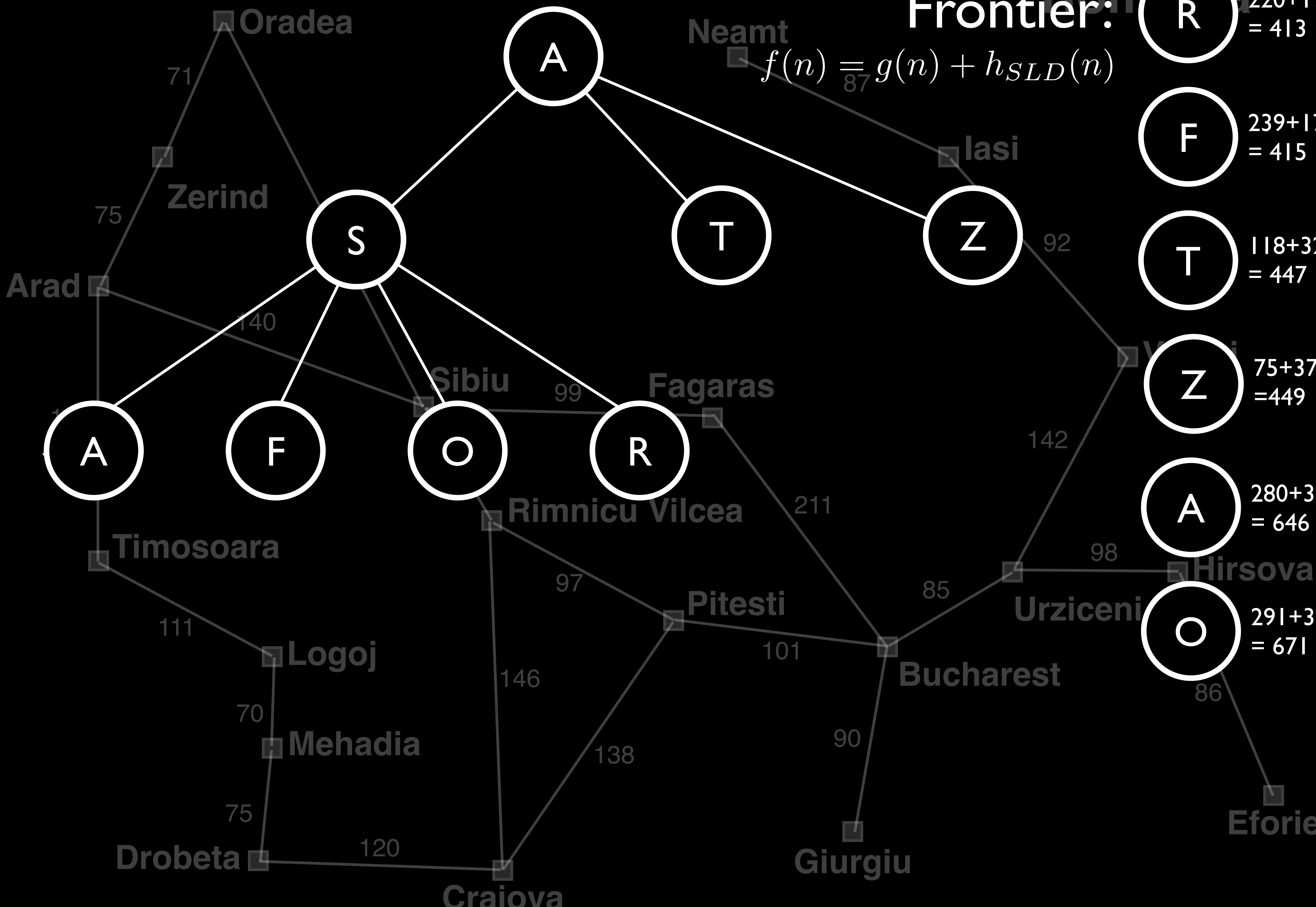
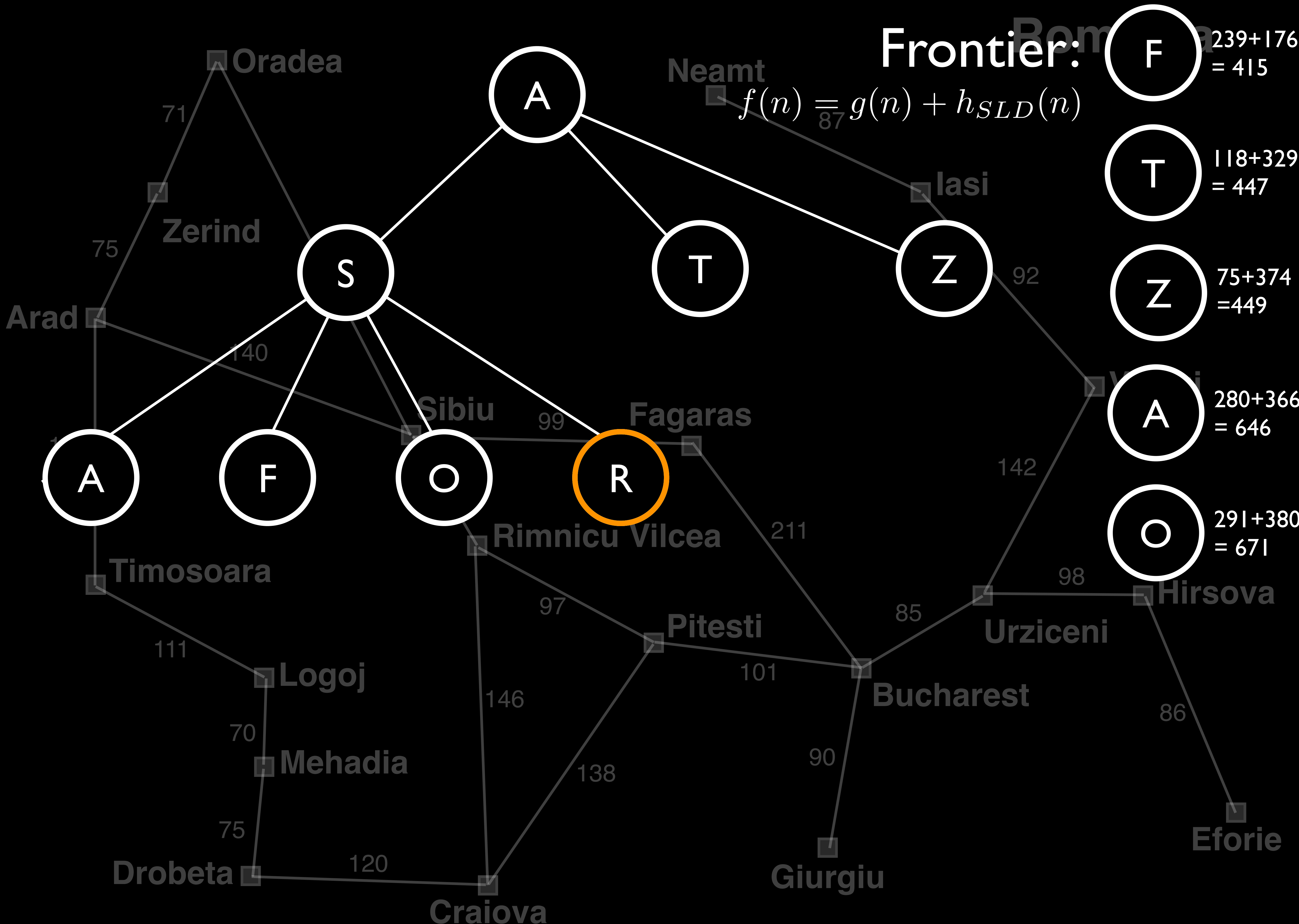
Frontier: $f(n) = g(n) + h_{SLD}(n)$

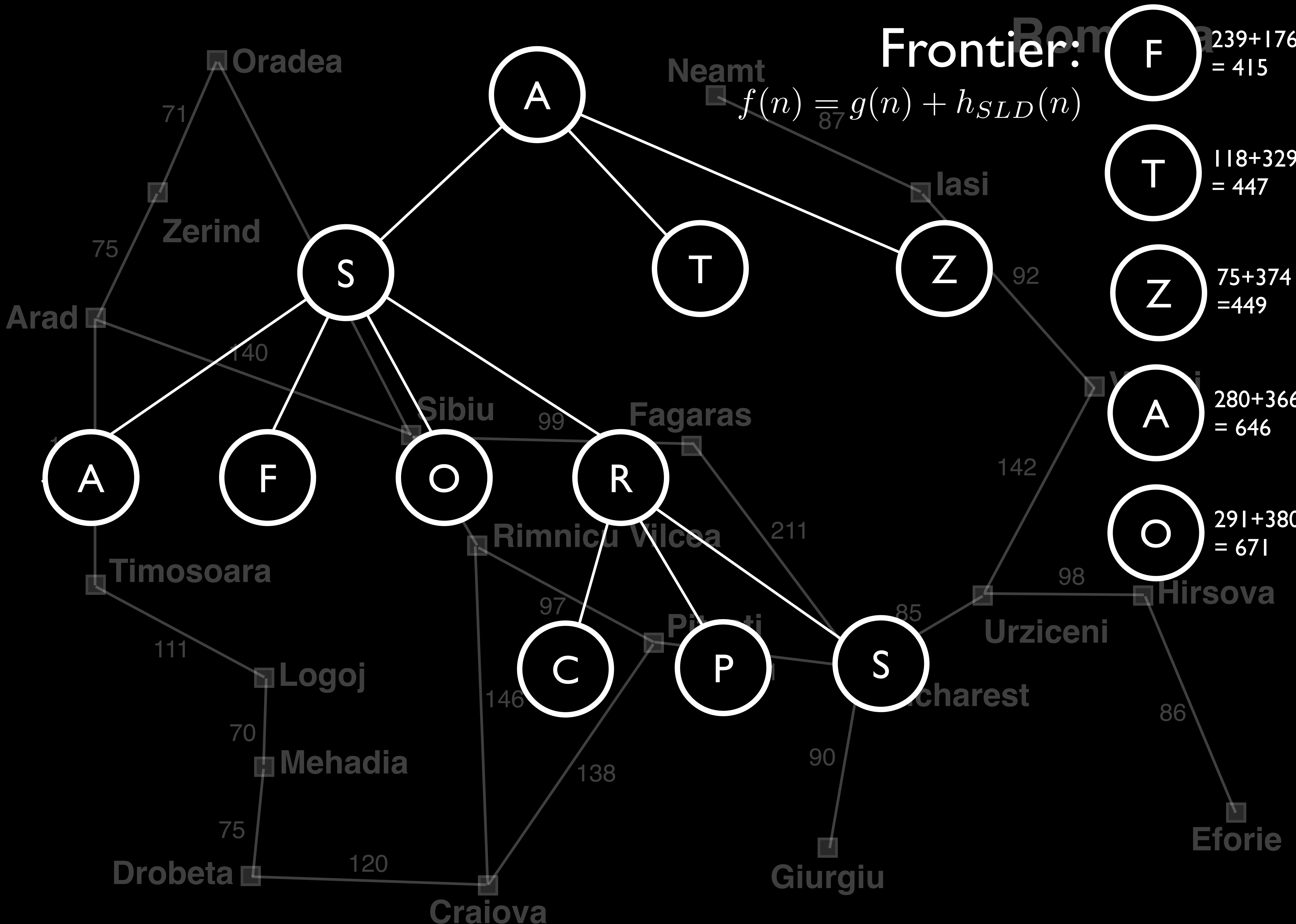



- R: 220+193 = 413
- F: 239+176 = 415
- T: 118+329 = 447
- Z: 75+374 =449
- A: 280+366 = 646
- O: 291+380 = 671

## Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$





Frontier:
$f(n) = g(n) + h_{SLD}(n)$
F 239+176 = 415
T 118+329 = 447
Z 75+374 =449
A 280+366 = 646
O 291+380 = 671
A
S
T
Z
A
F
O
R
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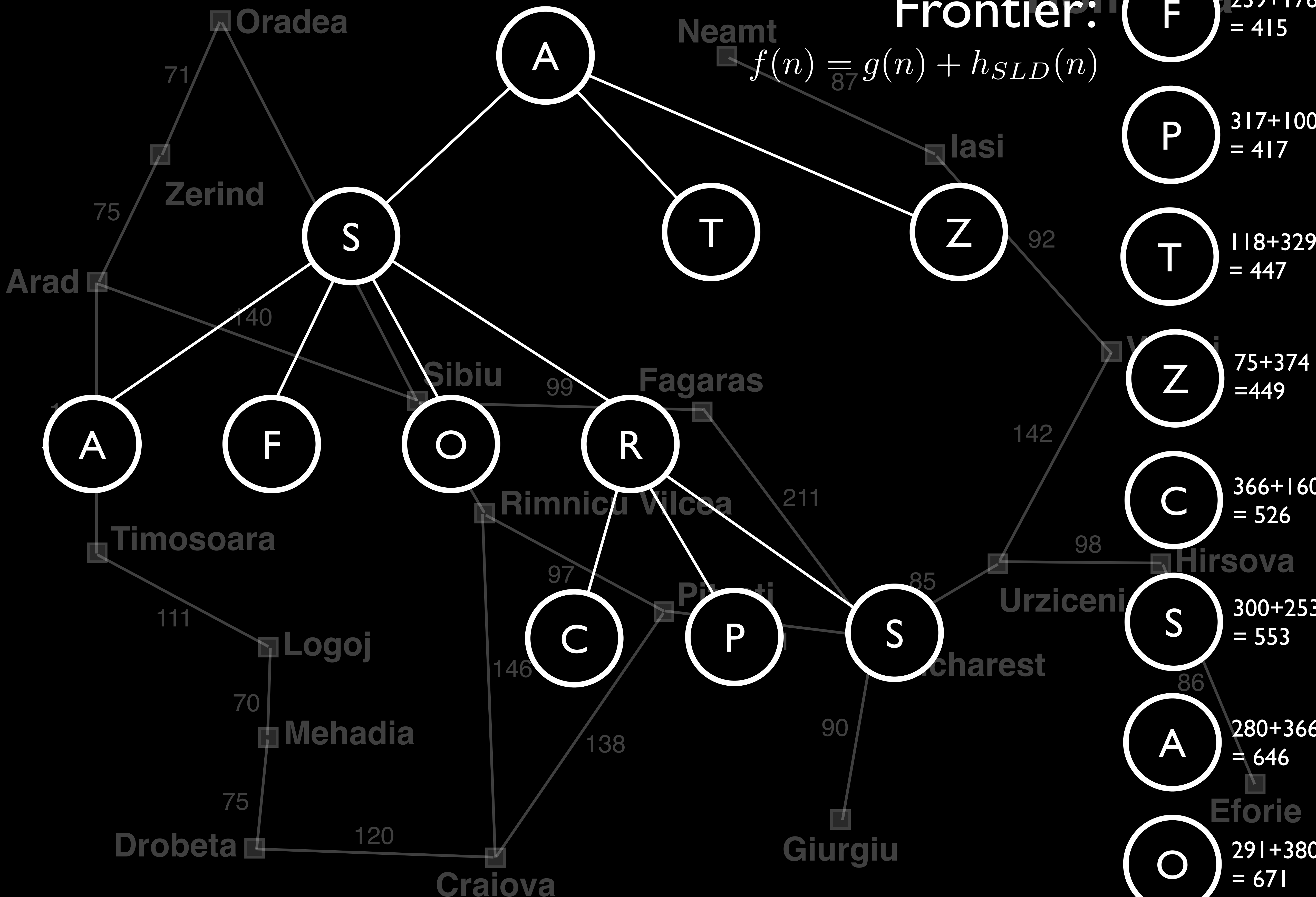
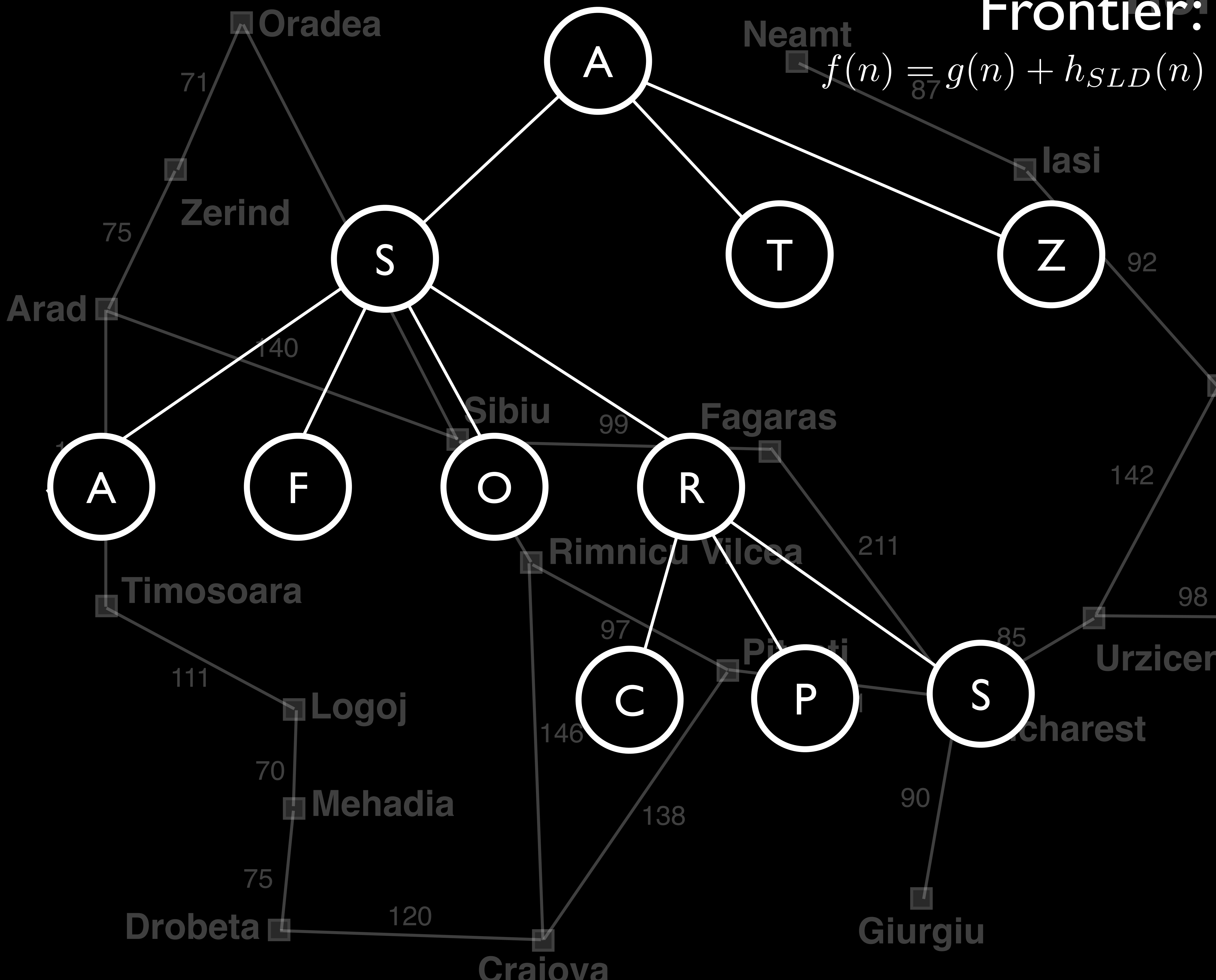
## Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$





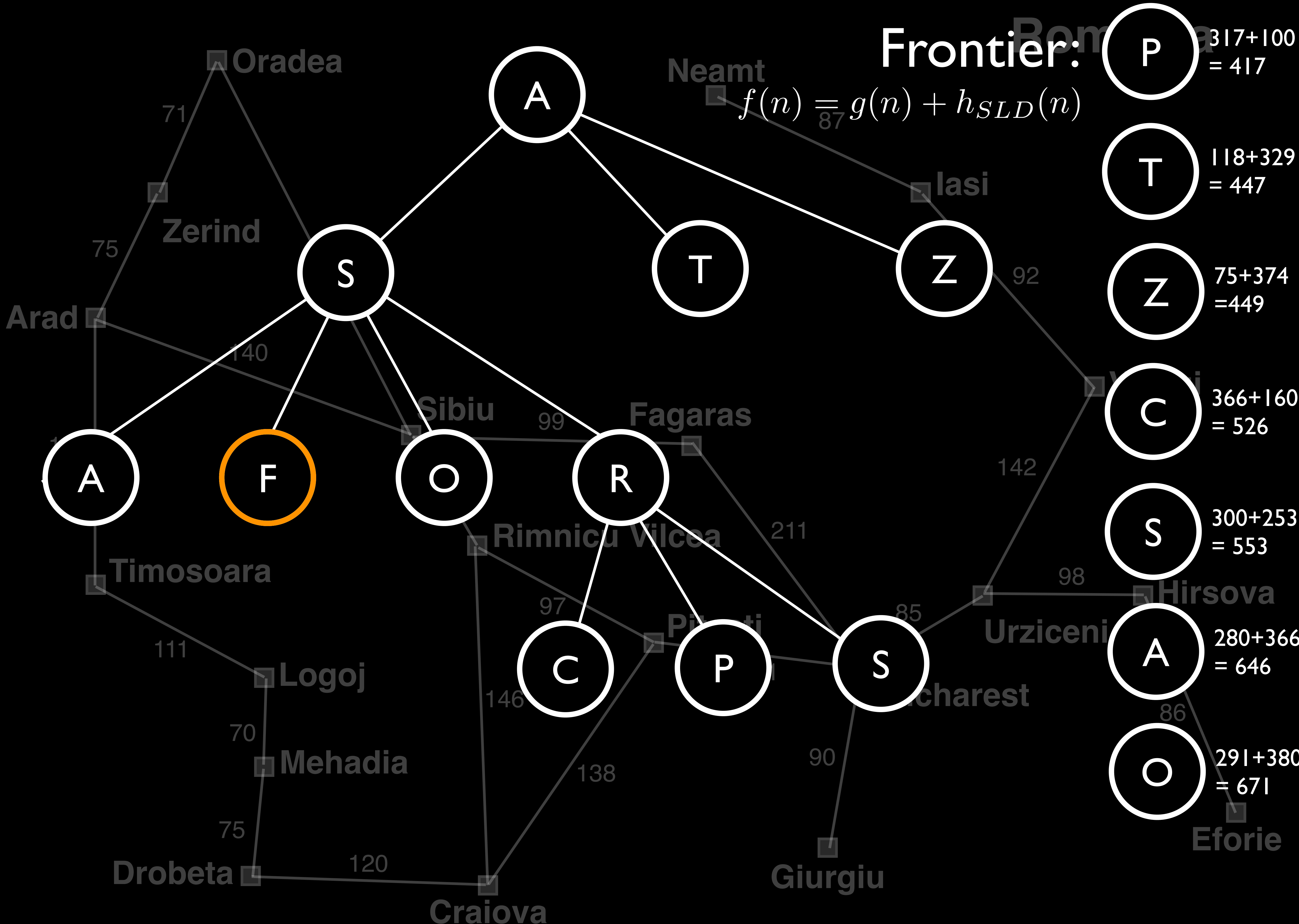

# Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$

- F: 239+176 = 415
- P: 317+100 = 417
- T: 118+329 = 447
- Z: 75+374 =449
- C: 366+160 = 526
- S: 300+253 = 553
- A: 280+366 = 646
- O: 291+380 = 671

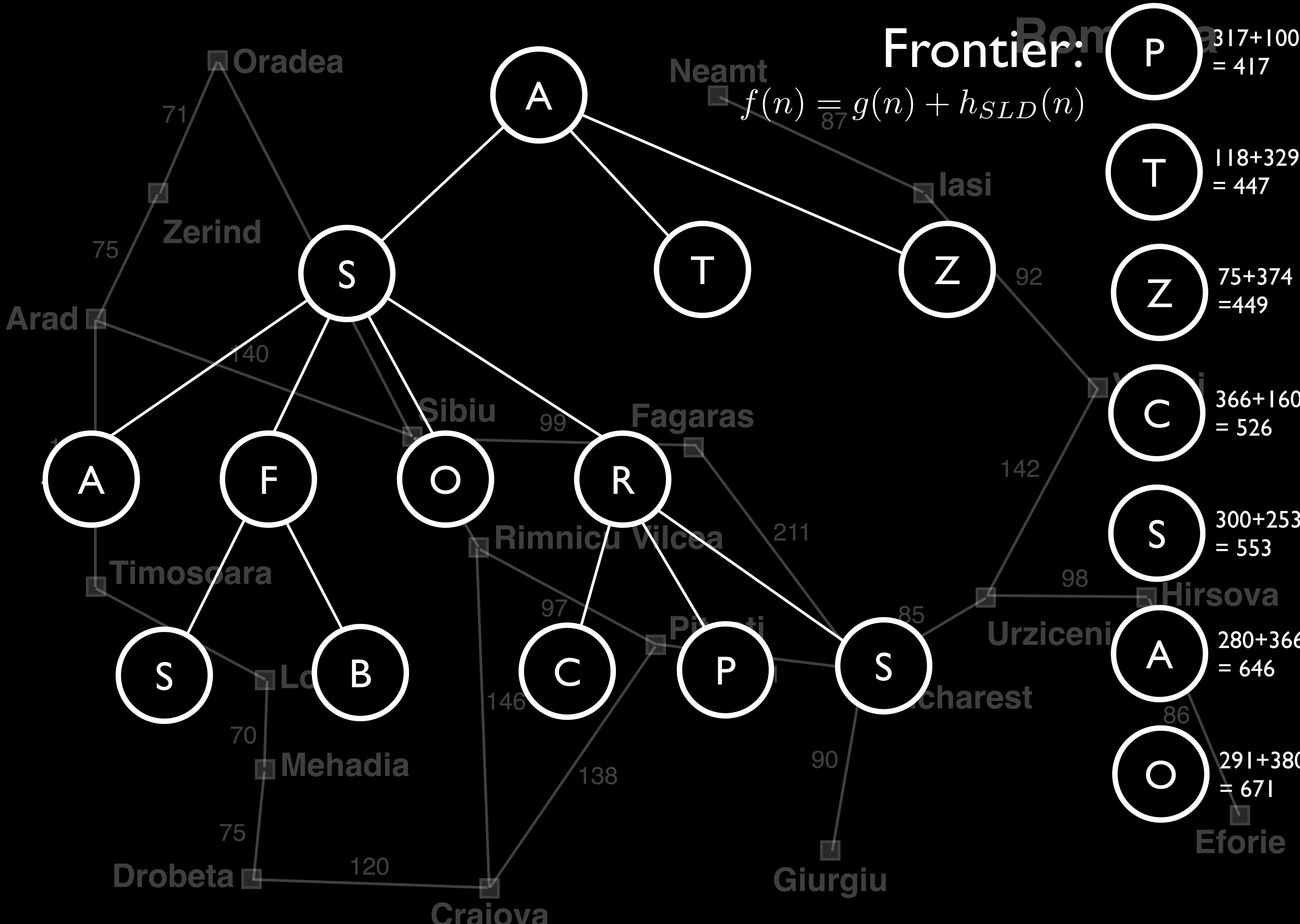
# Frontier:

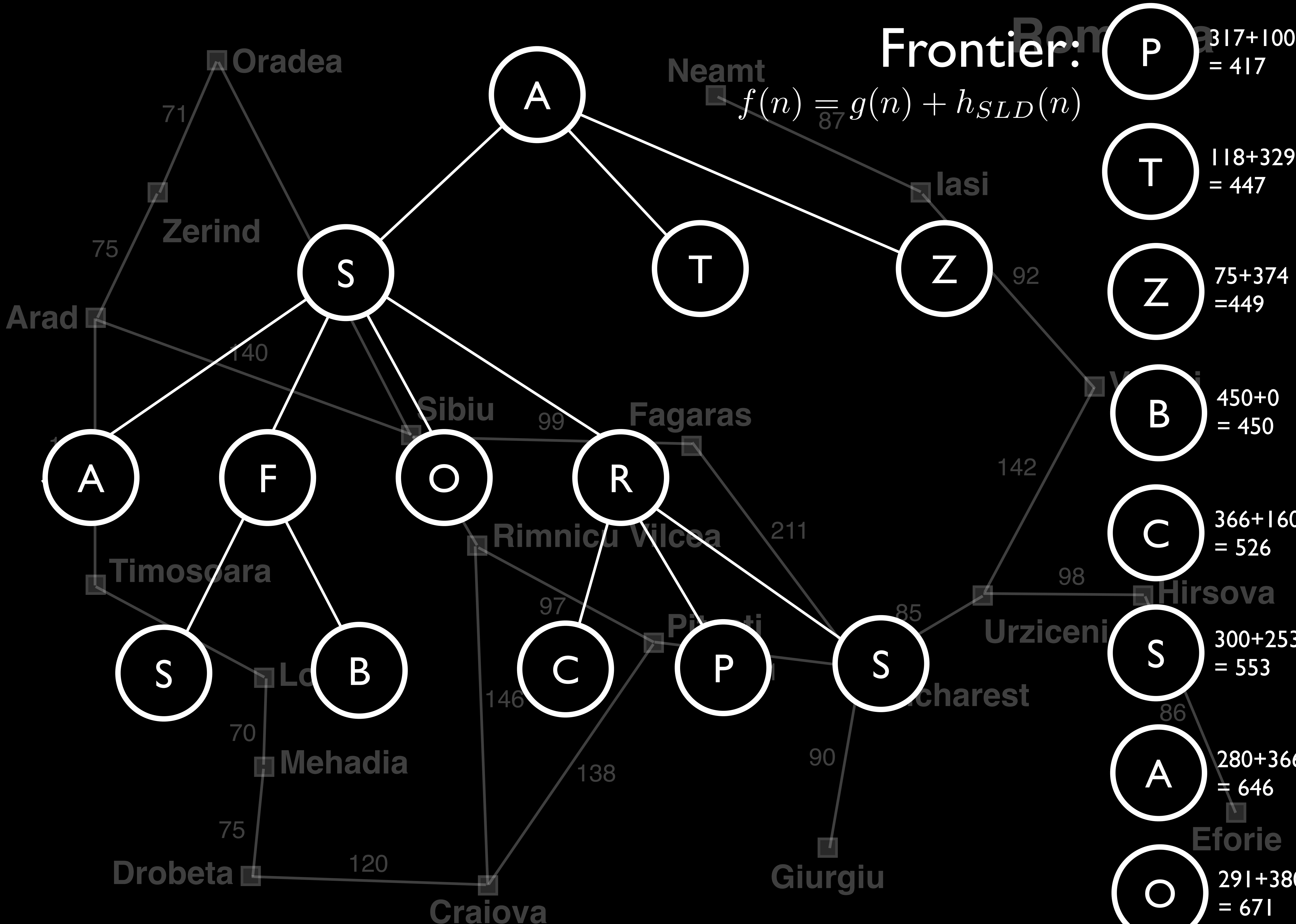
$$f(n) = g(n) + h_{SLD}(n)$$



# Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$

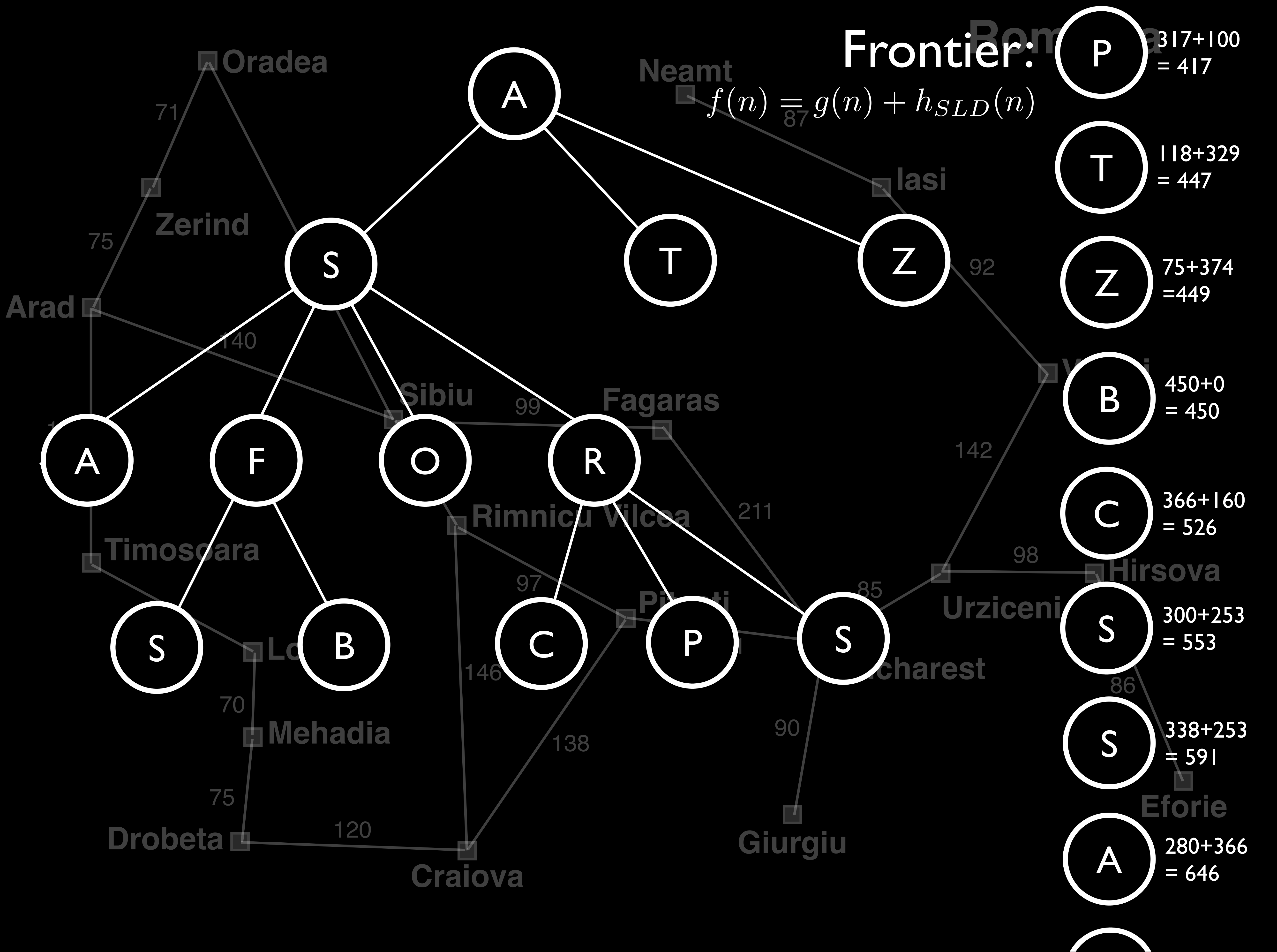




Frontier:
$f(n) = g(n) + h_{SLD}(n)$
A
S
T
Z
A
F
O
R
S
B
C
P
S
P
317+100
= 417
T
118+329
= 447
Z
75+374
=449
B
450+0
= 450
C
366+160
= 526
S
300+253
= 553
A
280+366
= 646
O
291+380
= 671
Oradea
Neamt
Iasi
Zerind
Arad
Sibiu
Fagaras
Rimnicu Vilcea
Urziceni
Hirsova
Mehadia
Drobeta
Craiova
Giurgiu
Eforie
71
75
87
92
140
99
142
211
97
98
85
146
70
90
138
86
75
120

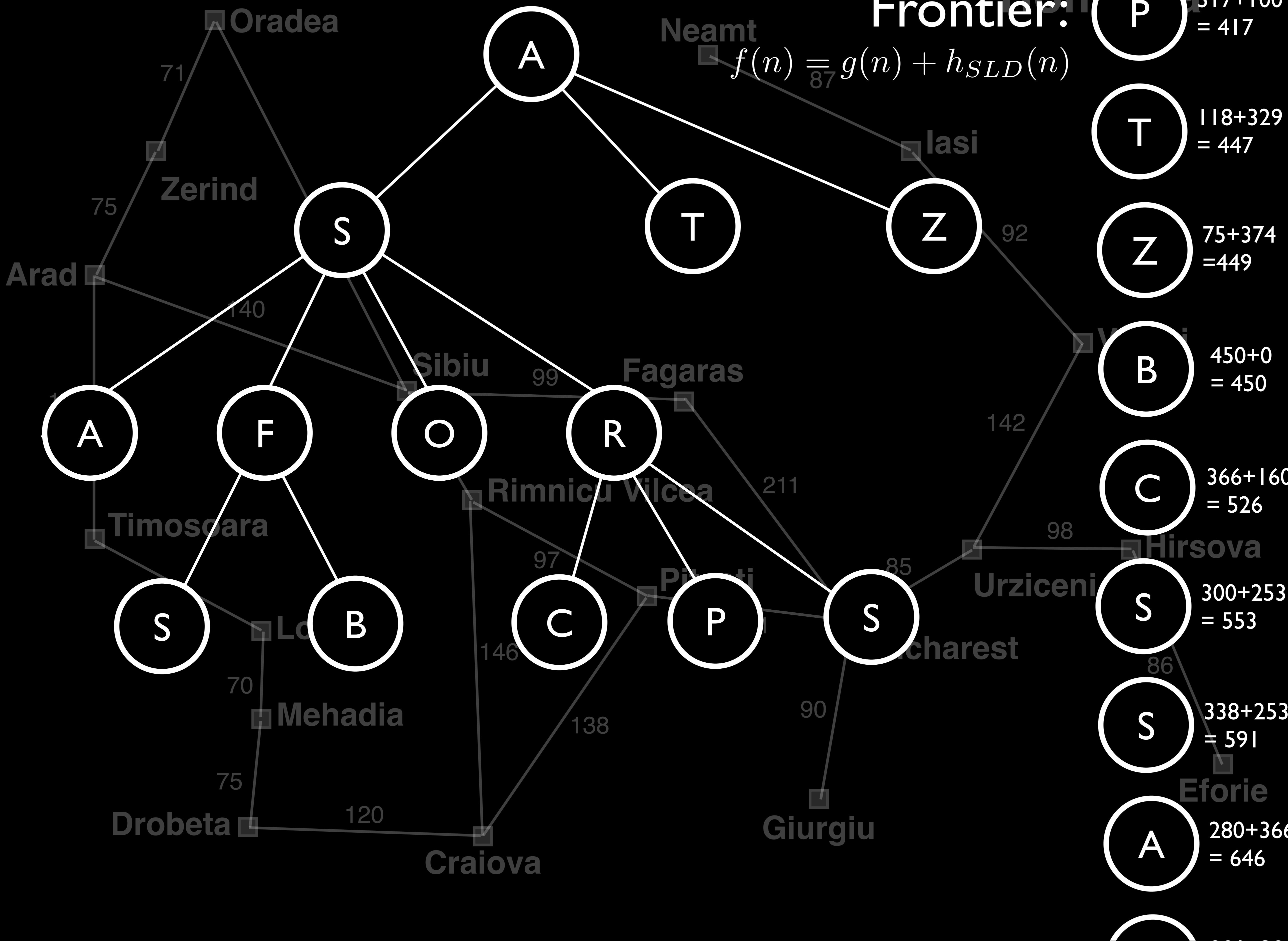
## Frontier:

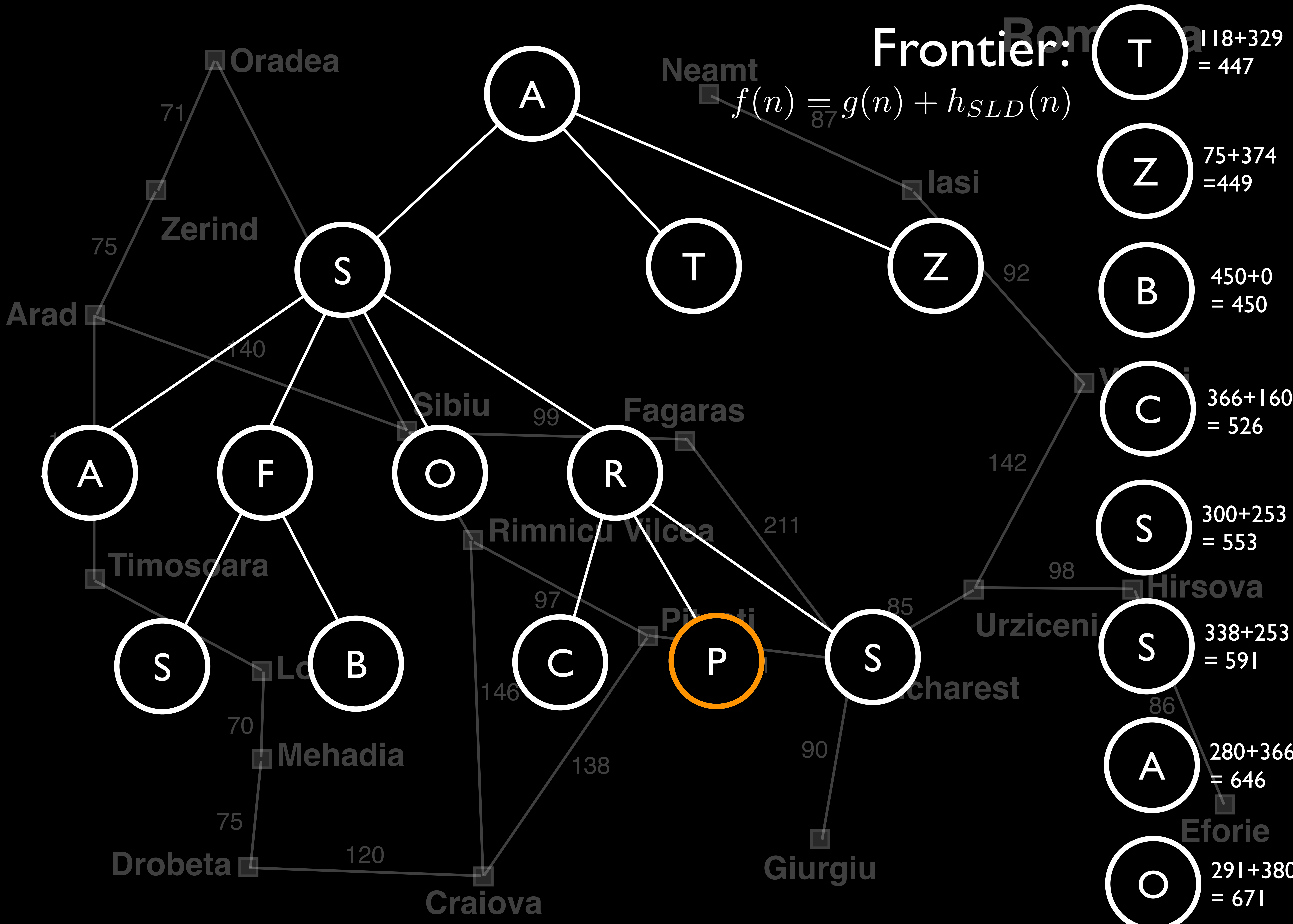
$$f(n) = g(n) + h_{SLD}(n)$$



# Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$




Frontier:
$f(n) = g(n) + h_{SLD}(n)$
T 118+329 = 447
Z 75+374 =449
B 450+0 = 450
C 366+160 = 526
S 300+253 = 553
S 338+253 = 591
A 280+366 = 646
O 291+380 = 671
A
S
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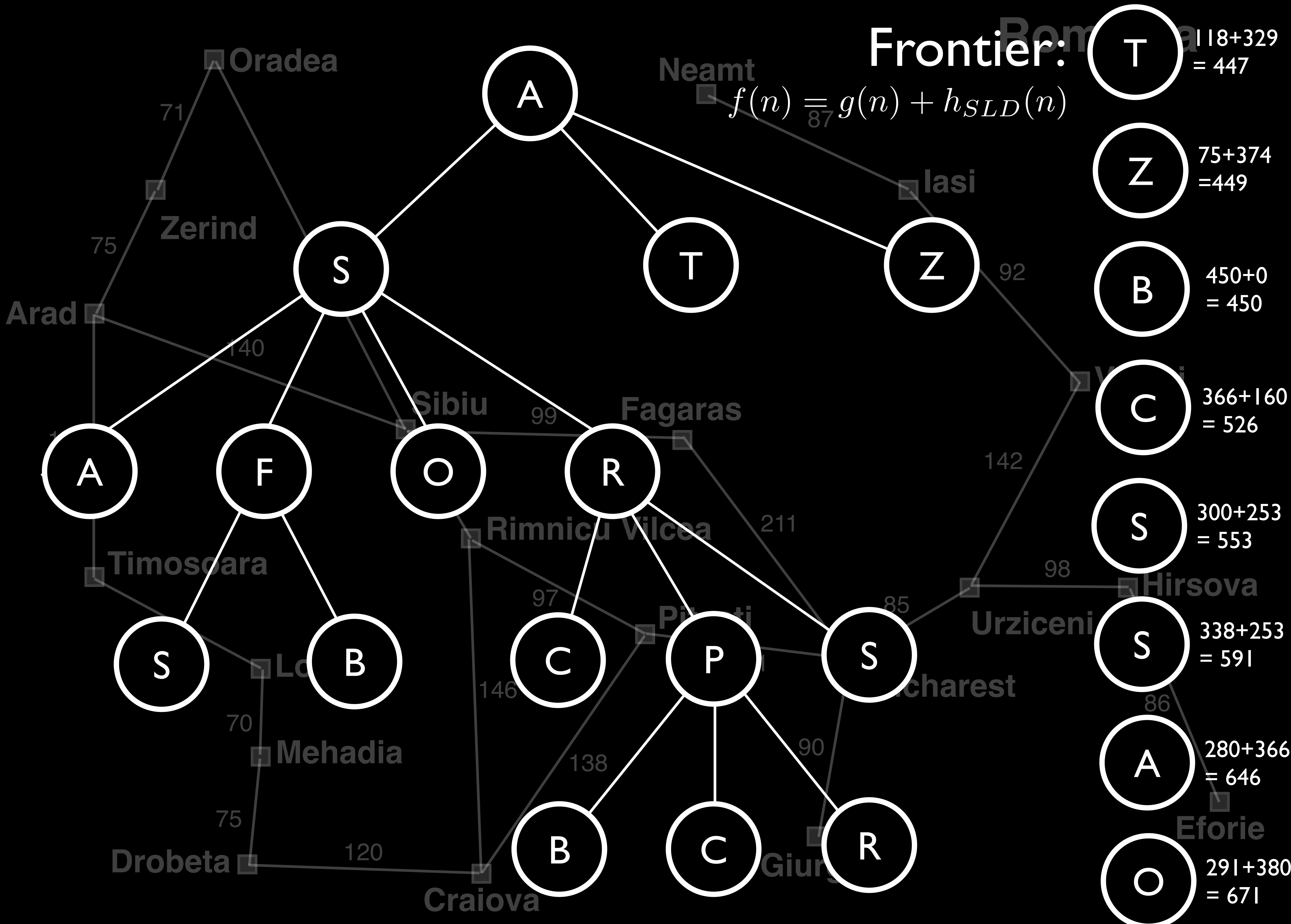
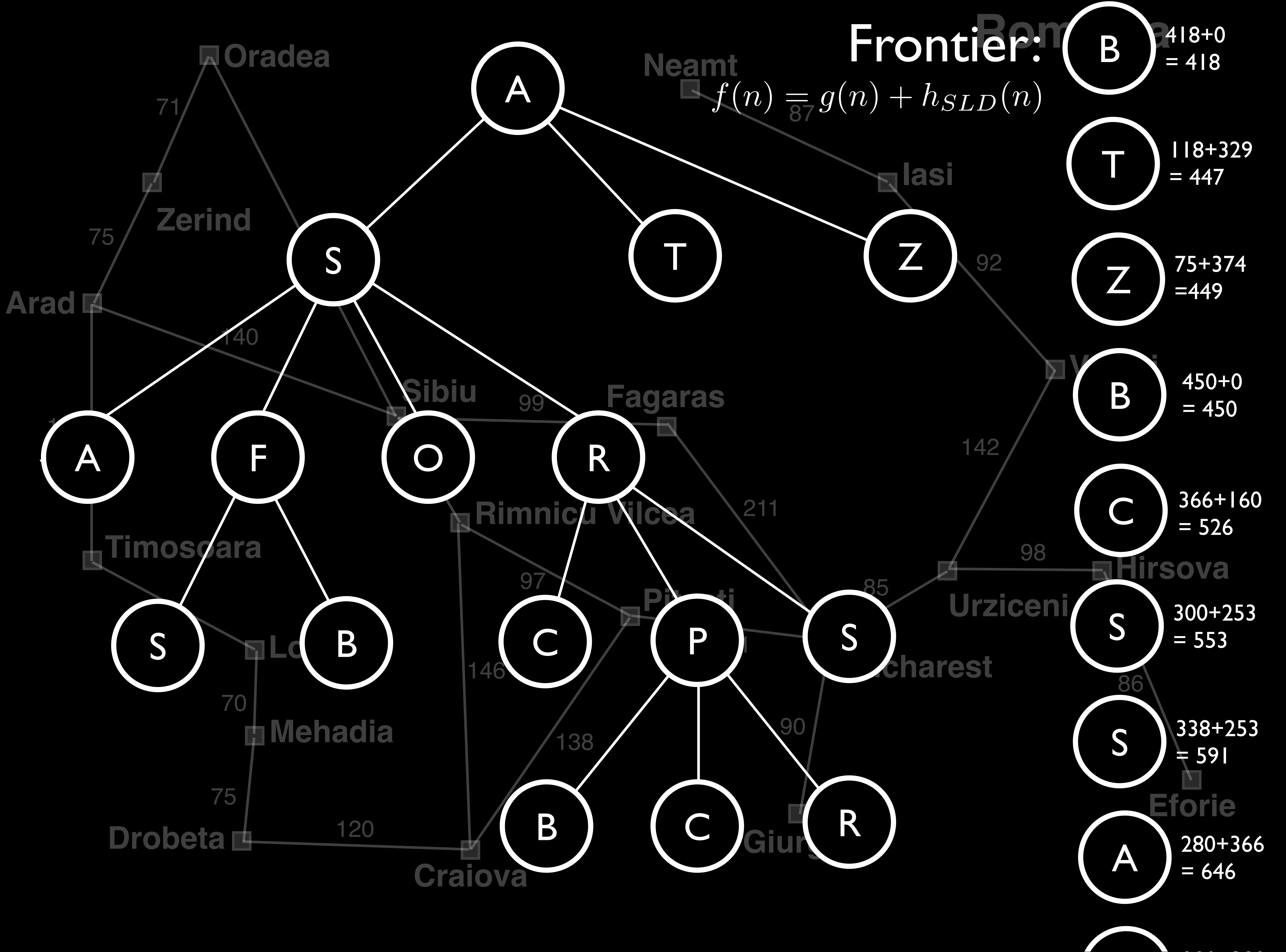
## Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$



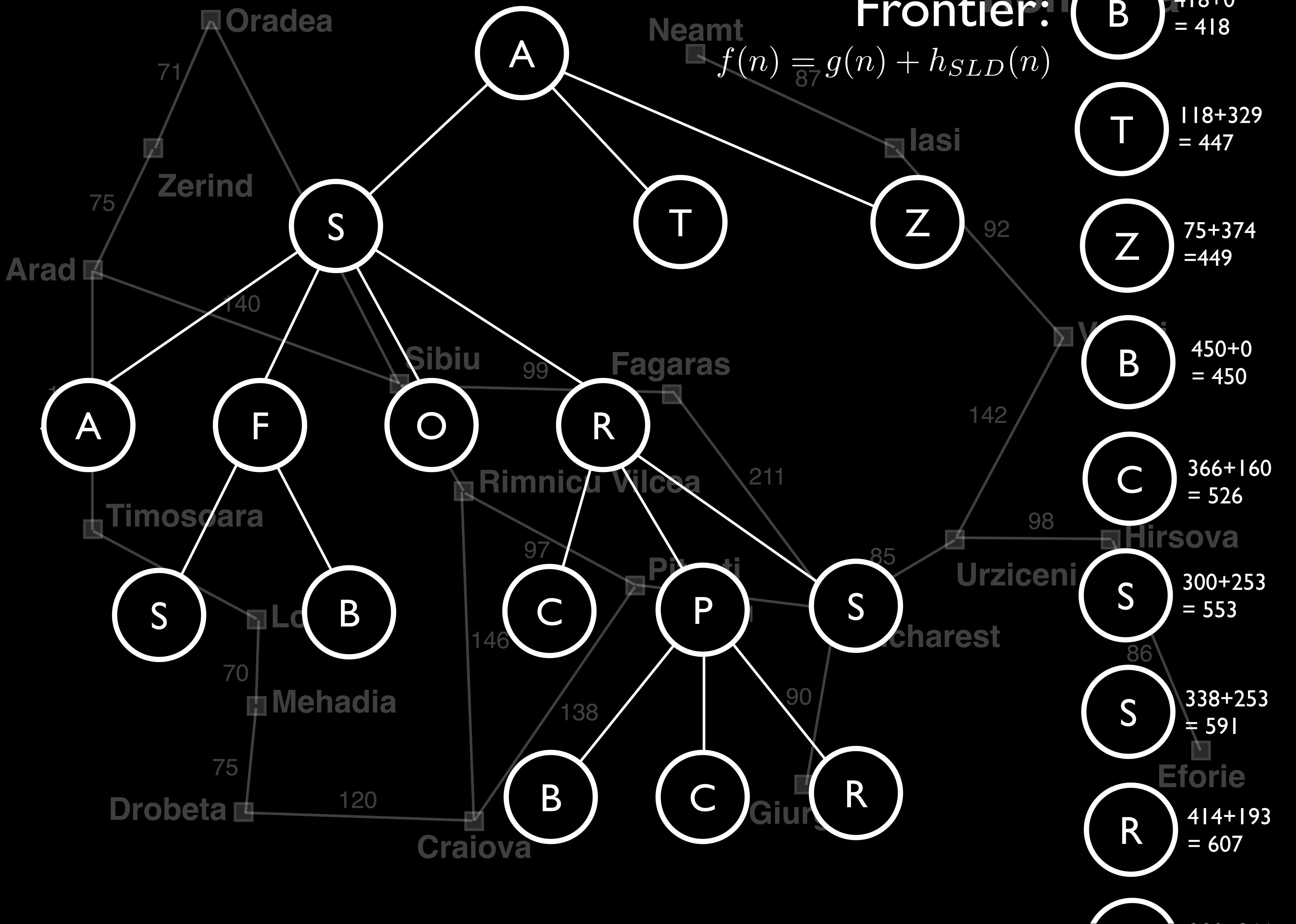
# Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$



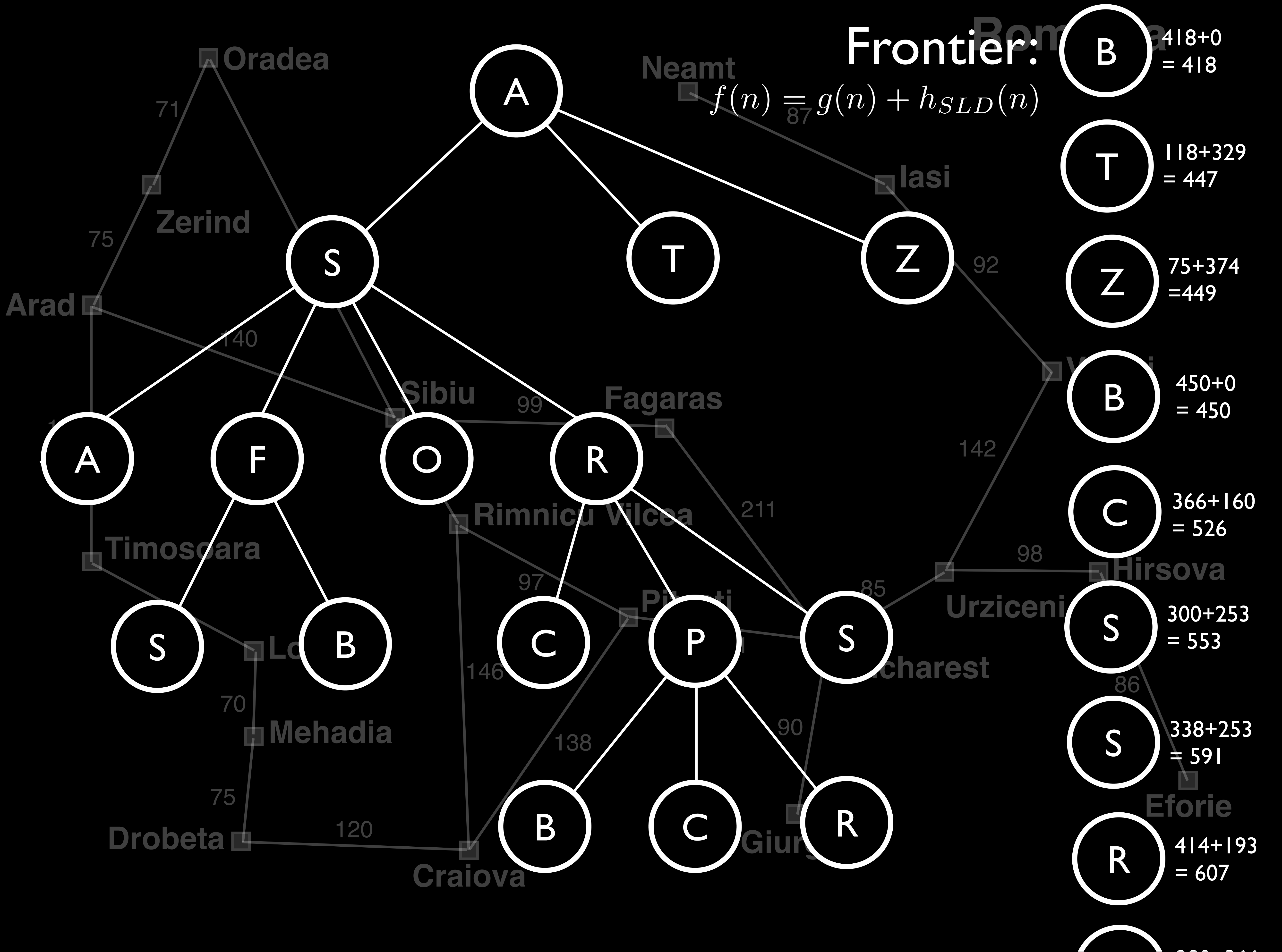
## Frontier:

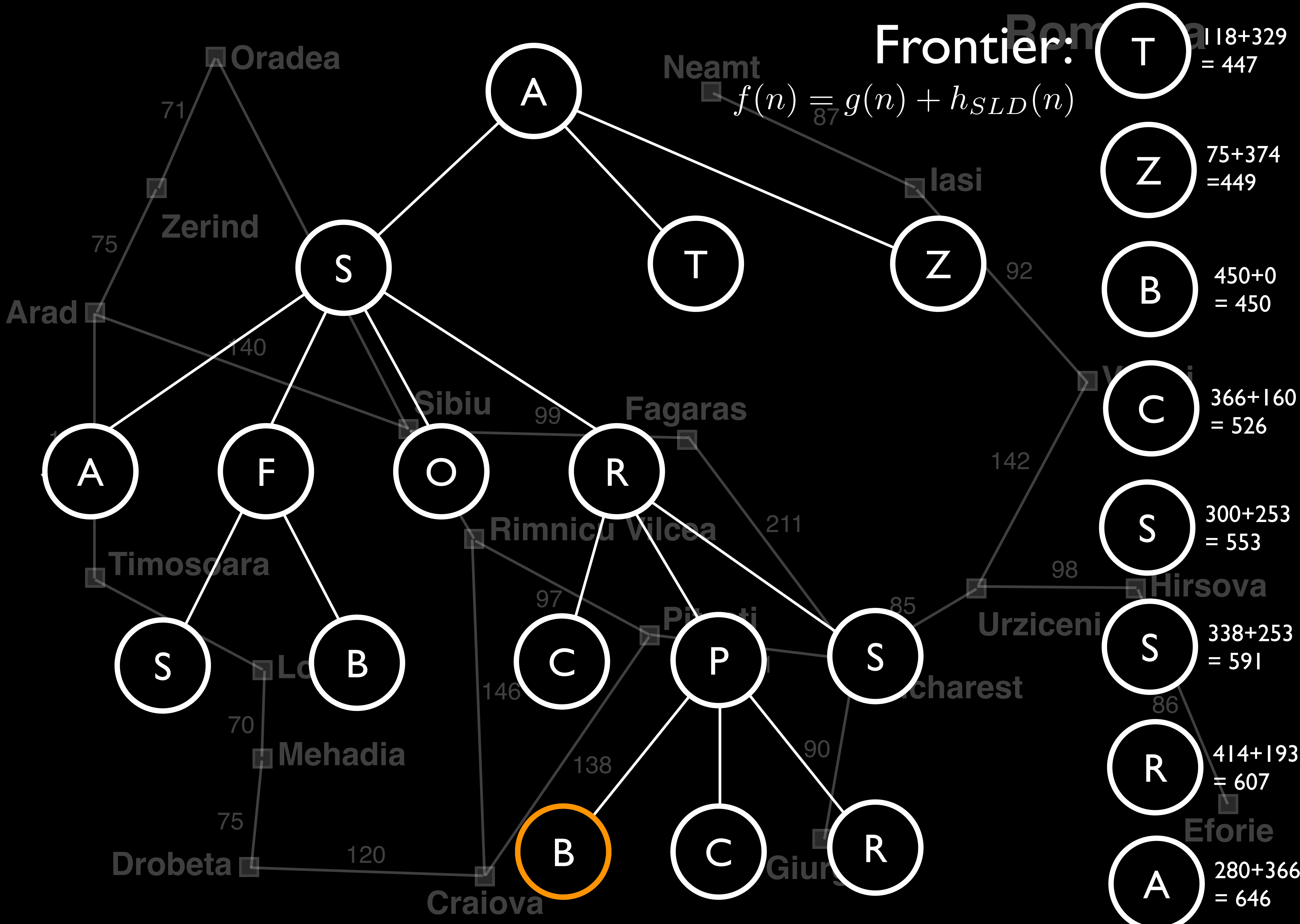
$$f(n) = g(n) + h_{SLD}(n)$$



Frontier:

$$f(n) = g(n) + h_{SLD}(n)$$




Frontier:
$f(n) = g(n) + h_{SLD}(n)$
T 118+329 = 447
Z 75+374 =449
B 450+0 = 450
C 366+160 = 526
S 300+253 = 553
S 338+253 = 591
R 414+193 = 607
A 280+366 = 646
A
S
T
Z
A
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B
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Oradea
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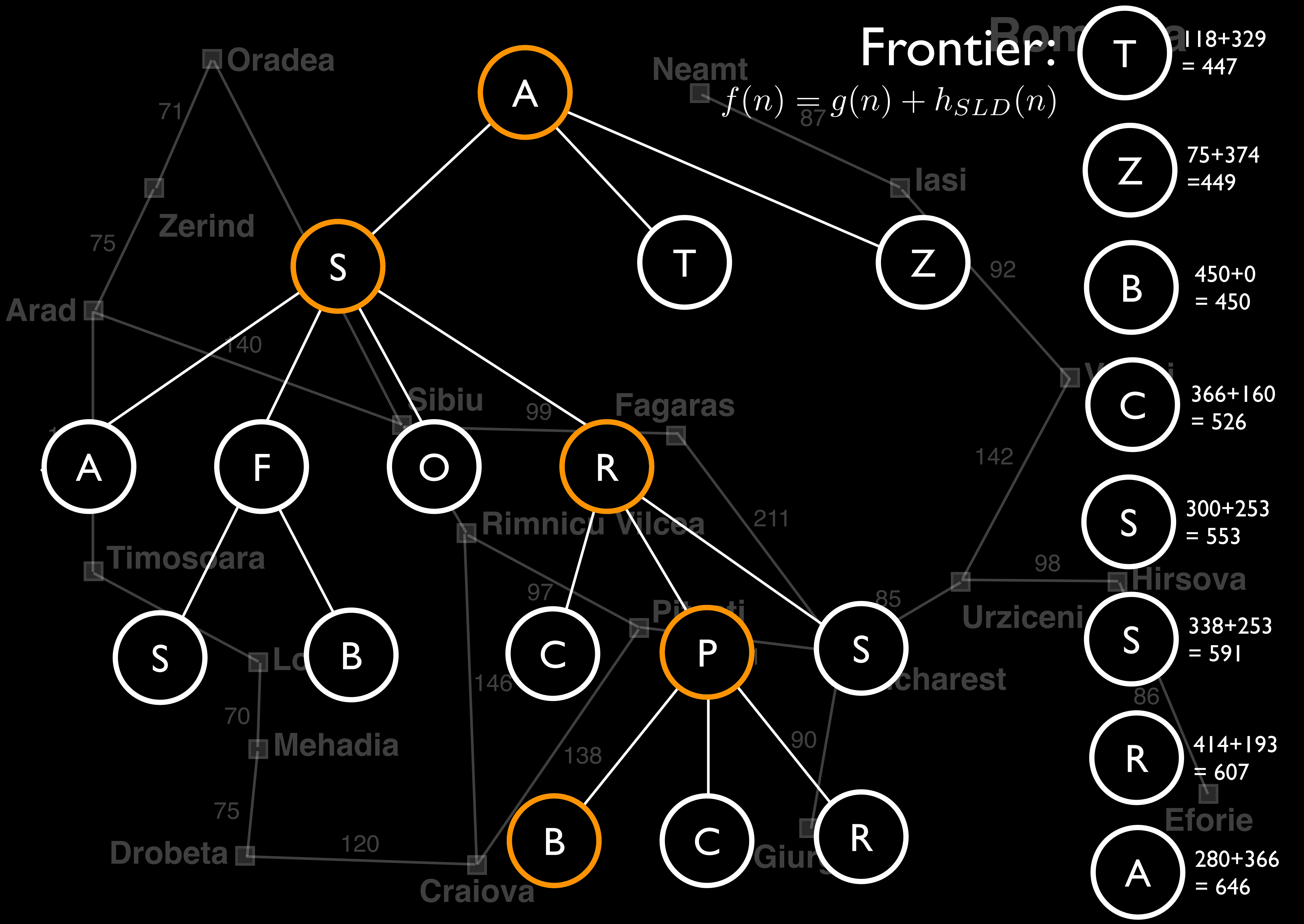
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Evaluation function

$$f(n) = g(n) + h(n)$$

Known cost of path from start node to node *n*

Estimated cost of cheapest path from *n* to a goal node

Evaluation function

$$f(n) = g(n) + h(n)$$

$=$ Estimated cost of cheapest solution through *n*

# A* Search

# A* Search

Completeness

Optimality

# A* Search

If $h(n)$
is admissible

Completeness

Optimality

# Admissible Heuristic

Never overestimates the true cost of a solution

# Admissible Heuristic

Never overestimates the true cost
of a solution

$$f(n) = g(n) + h_{SLD}(n)$$

# A* Search

If $h(n)$ is admissible

Completeness

Optimality

Time Complexity

Space Complexity

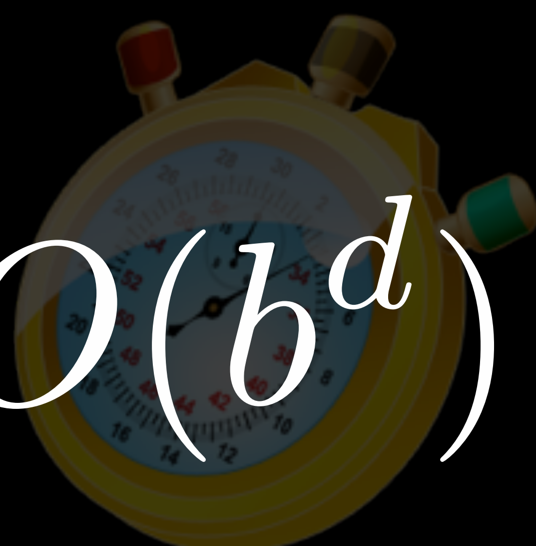
# A* Search

Completeness

If $h(n)$

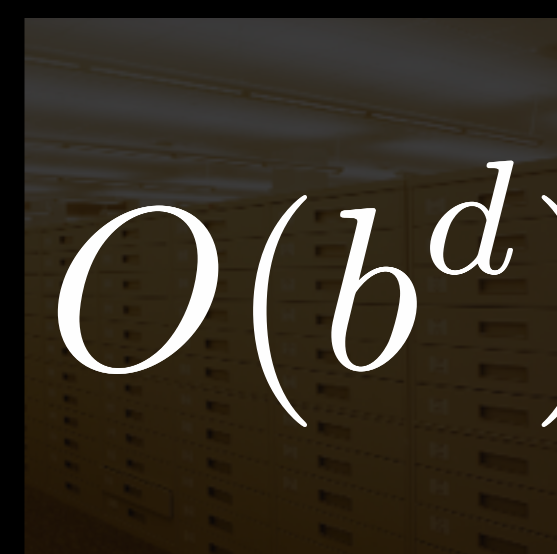
is admissible

Optimality



$O(b^d)$

Time Complexity



$O(b^d)$

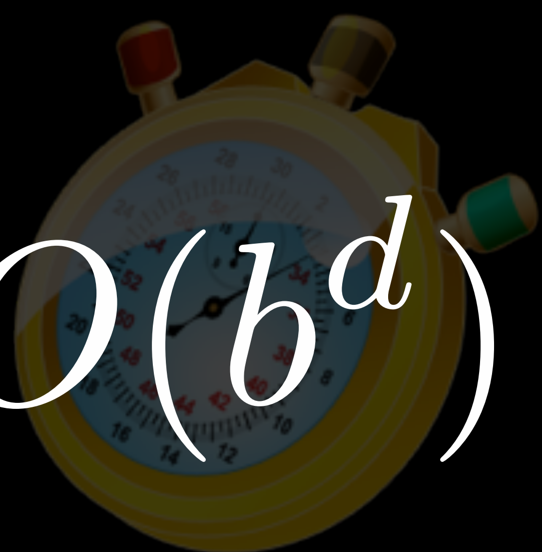
Space Complexity

# A* Search

If $h(n)$ is admissible

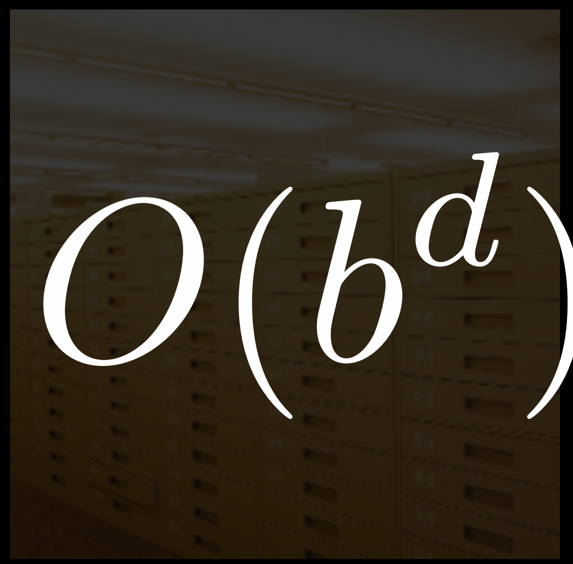
Completeness

Optimality



$O(b^d)$

?!?!?!



$O(b^d)$

Time Complexity

Space Complexity

# Time / Space Advantage of A*

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- A* does much better than its worst case complexity when the heuristic is close to the true shortest cost to a goal

# Time / Space Advantage of A*

- A* does much better than its worst case complexity when the heuristic is close to the true shortest cost to a goal
- Where h* is the optimal (perfect) heuristic, A* runs in provably polynomial time when

$$\left|h(x) - h^*(x)\right| = O(\log h^*(x))$$

Now It's Time to Play...

# Heuristic Functions

Where do good heuristic functions come from?

# Heuristic Functions

Where do good heuristic functions come from?

Good question...
(See Section 3.6 for some ideas)

# Search Strategies

| | BFS | DFS | IDS | Greedy | A* |
|---|---|---|---|---|---|
| Complete? | ✓ | ✗ | ✓ | ✗ | ✓ |
| Optimal? | ✓ | ✗ | ✓ | ✗ | ✓ |
| Time | $O(b^d)$ | $O(b^m)$ | $O(b^d)$ | $O(b^m)$ | $O(b^{\epsilon d})$ |
| Space | $O(b^d)$ | $O(bm)$ | $O(bd)$ | $O(b^m)$ | $O(b^d)$ |

* If step costs are identical

† With an admissible heuristic

Thursday: Homework Assignment - Check Website

Next Tuesday: Quiz