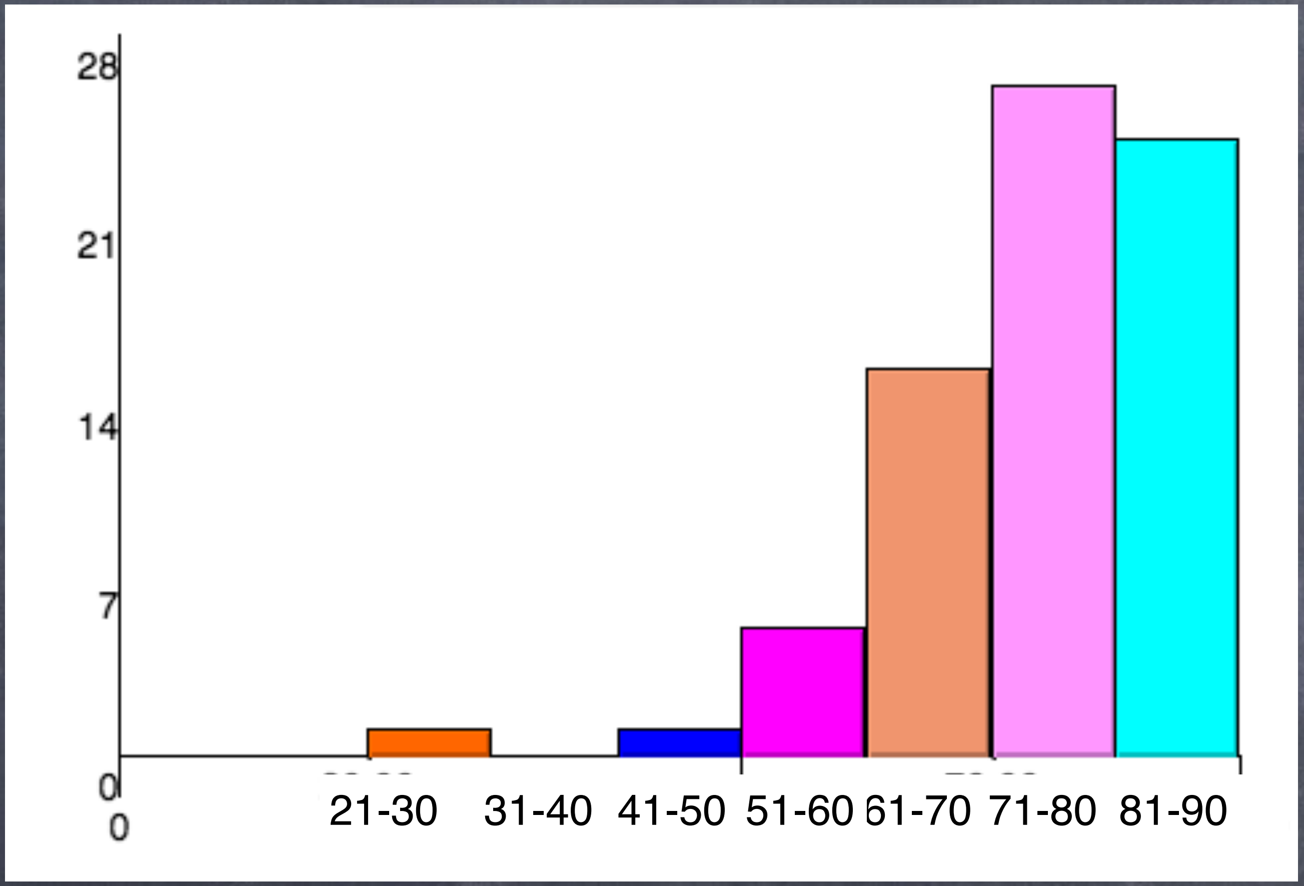
CSC242: Intro to AI

Lecture 10

Exam 1



ULW

- Detailed outline due March 4
  - HARD DEADLINE!
- One full page
- Include citations to at least 3 sources

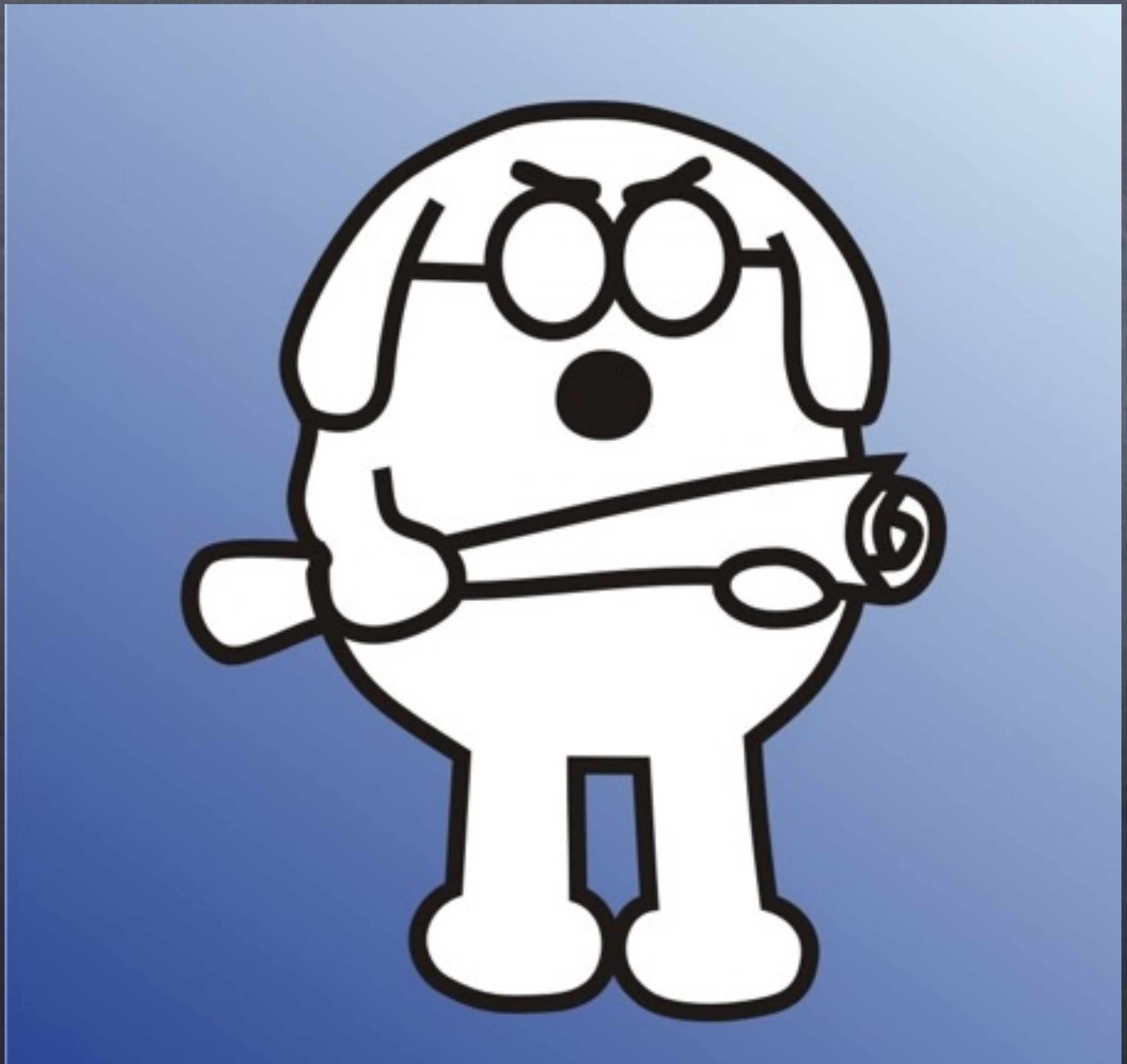
Othello

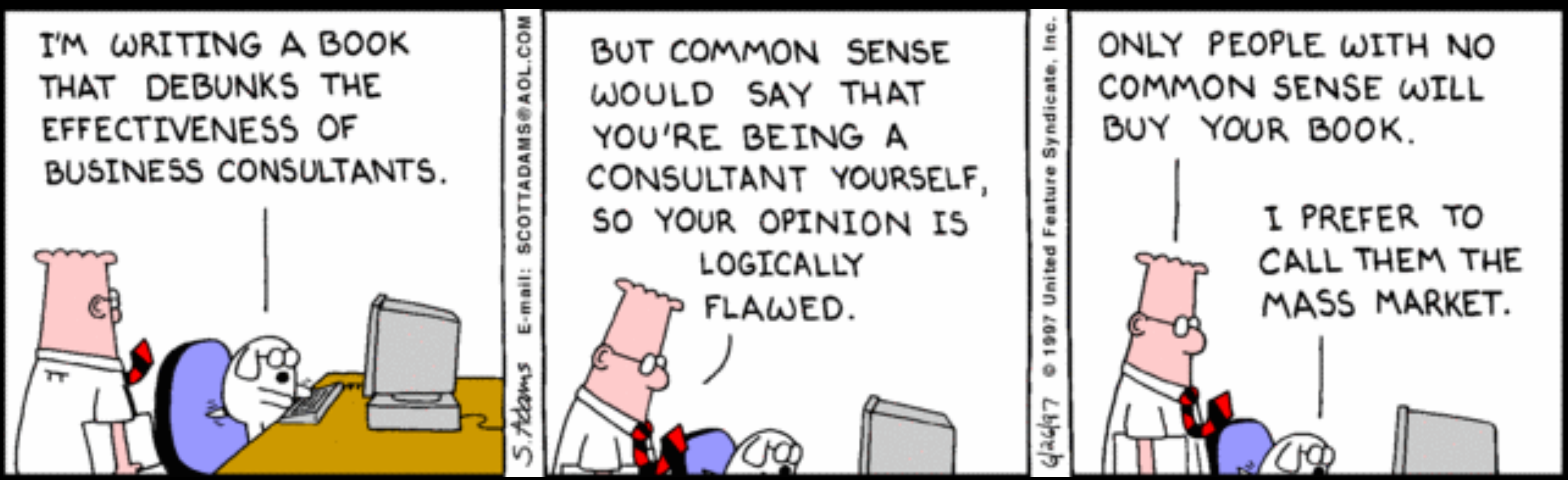
- Thursday Feb 27 by 11:59pm:
- One member of team (either one) upload via Blackboard zip file containing
  - Single directory, name of the form

screaming-banjos-tim-kopp-henry-kautz

  - Source code
  - Compiled code (linux)
  - Report (pdf, doc, docx)

First-Order Logic




I'M WRITING A BOOK THAT DEBUNKS THE EFFECTIVENESS OF BUSINESS CONSULTANTS.
S. Adams
E-mail: SCOTTADAMS@AOL.COM
BUT COMMON SENSE WOULD SAY THAT YOU'RE BEING A CONSULTANT YOURSELF, SO YOUR OPINION IS LOGICALLY FLAWED.
6/26/97
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ONLY PEOPLE WITH NO COMMON SENSE WILL BUY YOUR BOOK.
I PREFER TO CALL THEM THE MASS MARKET.

- Rooms adjacent to pits will have breezes
- Socrates is a person
  All people are mortal
- Anybody’s grandmother is either their mother’s or their father’s mother

Logic 2.0

- Define a language based on propositional logic that will allow us to say all these things
- Define entailment ("follows from")
- Find inference rules that will allow us to compute the consequences of our knowledge (entailments)

“Socrates is a person”

P socrates_is_a_person

"Socrates is a person"

	P	socrates_is_a_person
"George is a person"	Q	george_is_a_person
"Fido is a dog"	R	fido_is_a_dog
"Dogbert is a dog"	S	dogbert_is_a_dog
"Dogbert is an executive"	T	dogbert_is_an_executive

Object-Oriented

- Objects: Socrates, George, Fido, Dogbert
- Classes: Person, Dog, Executive, ...

```
Socrates = new Person()
George = new Person()
Fido = new Dog()
Dogbert = new (Dog() and Executive())?
```

Constants

- Symbols denoting objects in the world
- Socrates, George, Fido, Dogbert, ...

denote |diˈnōt|

verb [trans.]
be a sign of; indicate : *this mark denotes purity and quality.*
• (often **be denoted**) stand as a name or symbol for : *the level of output per firm, denoted by X.*

Predicates

- Denote relationships between objects
- *Predicate(arg1, arg2, ..., argn)*
  - Man(Socrates), Dog(Fido)
  - Adjacent(George, Fido)
  - Between(Cheese, Burger, Bun)
  - Dog(Dogbert) and Executive(Dogbert)

“Anybody’s grandmother is either their mother’s or their father’s mother”

Functions

- Names for objects built out of other objects
- *function(arg1, arg2, ..., argn)*
  - *boss(George)*
  - *mother(Socrates)*
  - *room(1,2)*

- “George’s father is Dogbert’s boss”
  father(George) = boss(Dogbert)
- “Socrates’ mother is a person”
  Person(mother(Socrates))
- “Room 1,2 is adjacent to to room 1,1”
  Adjacent(room(1,2), room(1,1))

- Constants that denote objects in the world
- Function symbols that denote a mapping from a tuple of objects to another object
- Predicate symbols that denote relations (sets of tuples of objects)
- Term = constant or function+terms

Connectives

- Negation (not, $\neg$)
- Conjunction (and, $\wedge$)
- Disjunction (or, $\vee$)
- Implication (if-then, $\Rightarrow$)
- Biconditional (if and only if, $\Leftrightarrow$)

- Person(George) $\wedge$ Dog(Fido)
- Cat(Dogbert) $\wedge$ Dog(Dogbert)
- $\neg$ Mouse(Dogbert)
- Adjacent(room(2,1), room(1,1)) $\wedge$ Adjacent(room(2,1), room(2,2)) $\wedge$ ...
- Dog(Dogbert) $\Rightarrow$ Mammal(Dogbert)
- Dog(Dogbert) $\Rightarrow$ Barks(Dogbert)

- $Dog(Dogbert) \Rightarrow Mammal(Dogbert)$
- $Dog(Dogbert) \Rightarrow Barks(Dogbert)$
- $Dog(Scooby) \Rightarrow Mammal(Scooby)$
- $Dog(Scooby) \Rightarrow Barks(Scooby)$

- “All dogs are mammals”
- “All dogs bark”
- “All people are mortal”
- “All rooms adjacent to pits are breezy”

Variables

- Denote objects, but we don't know which
- Are terms: can appear wherever a constant or function expression can
    - Dog(x)
    - Happy(father(y))
    - Adjacent(room(x,y), room(+(x,1),y))

Quantifiers

- If α is a sentence, then so are
  - $\forall x\ \alpha$
  - $\exists x\ \alpha$

Universal quantification

- $\forall x\ \alpha$ is true if α is true for <u>all</u> possible values of x
- $\forall x\ Dog(x) \Rightarrow Mammal(x)$
- $\forall y\ Dog(y) \wedge Mammal(y)$

Existential quantification

- $\exists x\ \alpha$ is true if α is true for <u>some</u> possible values of x
- $\exists x$ Manages(x, Dogbert)
- $\exists r$ Room(r) $\wedge$ In(Wumpus, r)

First-Order Logic

- Constants and variables that denote objects in the world
- Function symbols that denote a mapping from a tuple of objects to another object
- Predicate symbols that denote relations (sets of tuples of objects)
- Connectives
- Quantifiers

A Programming Language for Knowledge

- Syntax:
  - What counts as a well-formed statement, formula, sentence, or program
- Semantics:
  - What these statements, formulas, sentences, or programs mean

Semantics of First-Order Logic

Models for PL

- Assignment of truth values to propositional variables

Models for PL

B	P	P	P	B
true	true	true	true	true
false	true	true	true	false
true	false	true	true	true
false	false	true	true	false
true	true	false	true	true
false	true	false	true	false
true	false	false	false	false
false	false	false	false	true

First-Order Logic

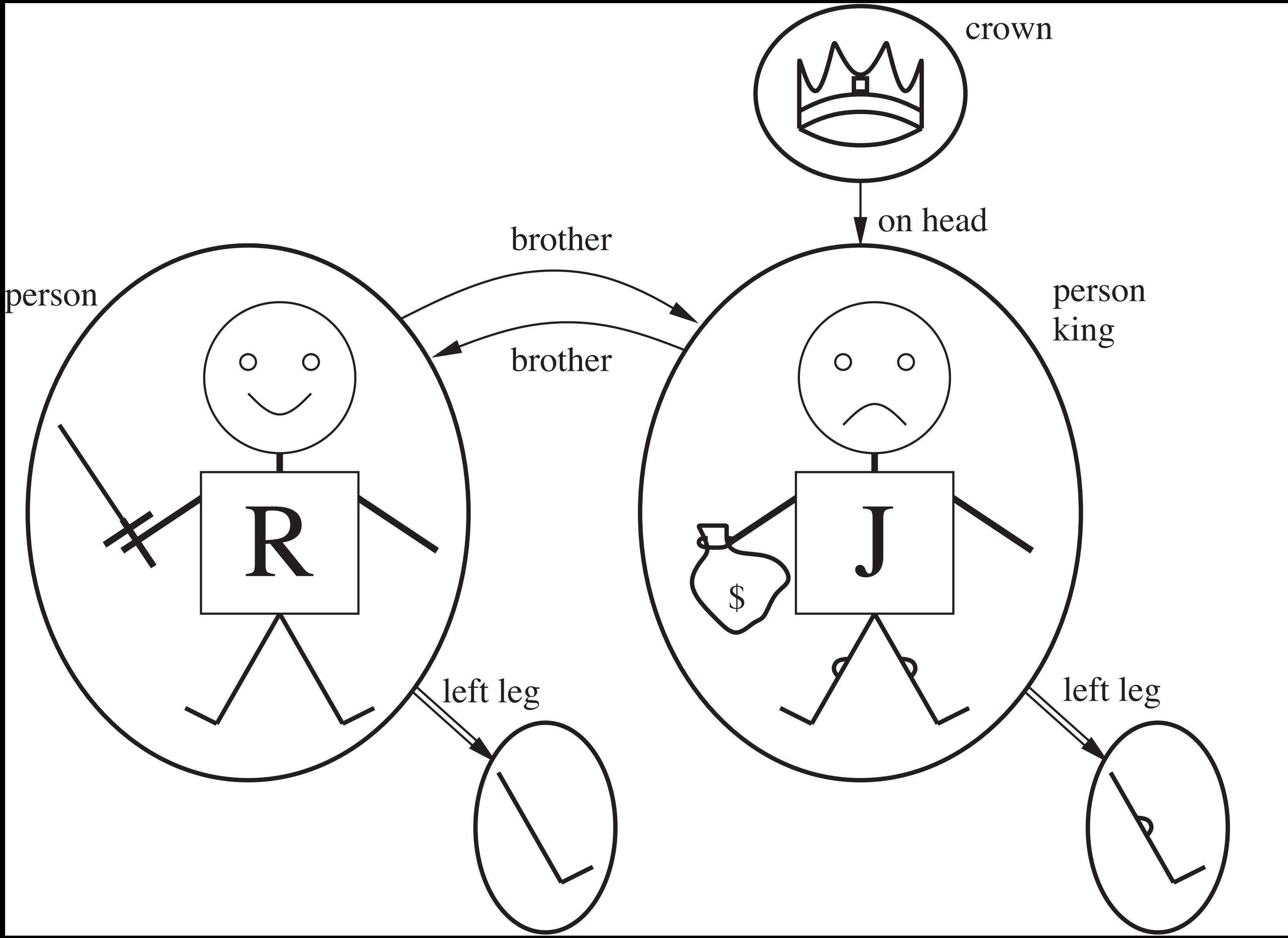
- Constants and variables that denote objects in the world
- Function symbols that denote a mapping from a tuple of objects to another object
- Predicate symbols that denote relations (sets of tuples of objects)
- Connectives
- Variables and Quantifiers

FOL: Constants

- Refer to objects in the world

Richard
(1157-1199)

John
(1166-1216)


crown
on head
brother
brother
person
person
king
R
J
$
left leg
left leg

Models for FOL: Domain

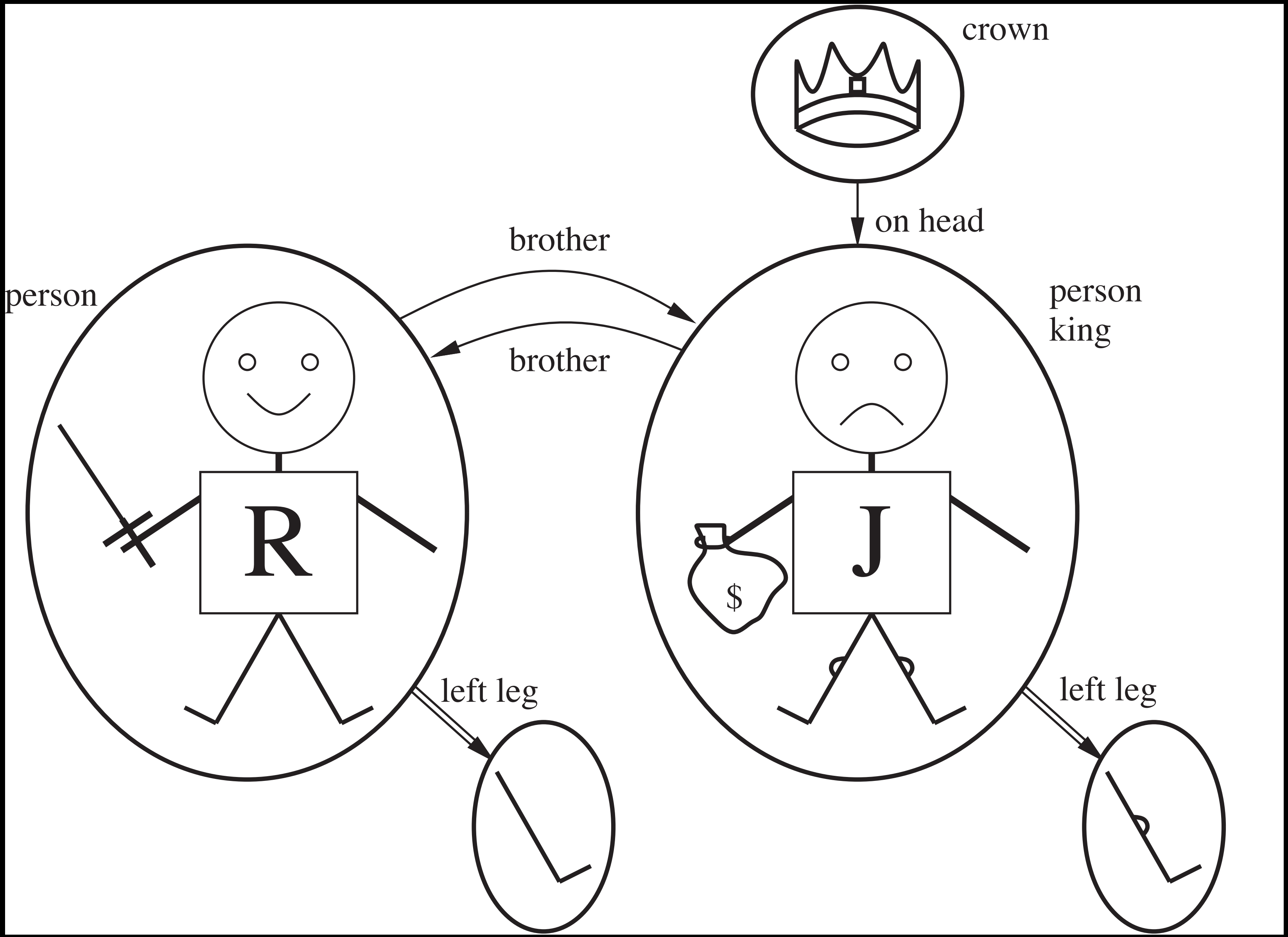
- Non-empty set of objects in the world

FOL: Constants

- Each constant refers to some element of the domain (some object in the world)

FOL: Predicates

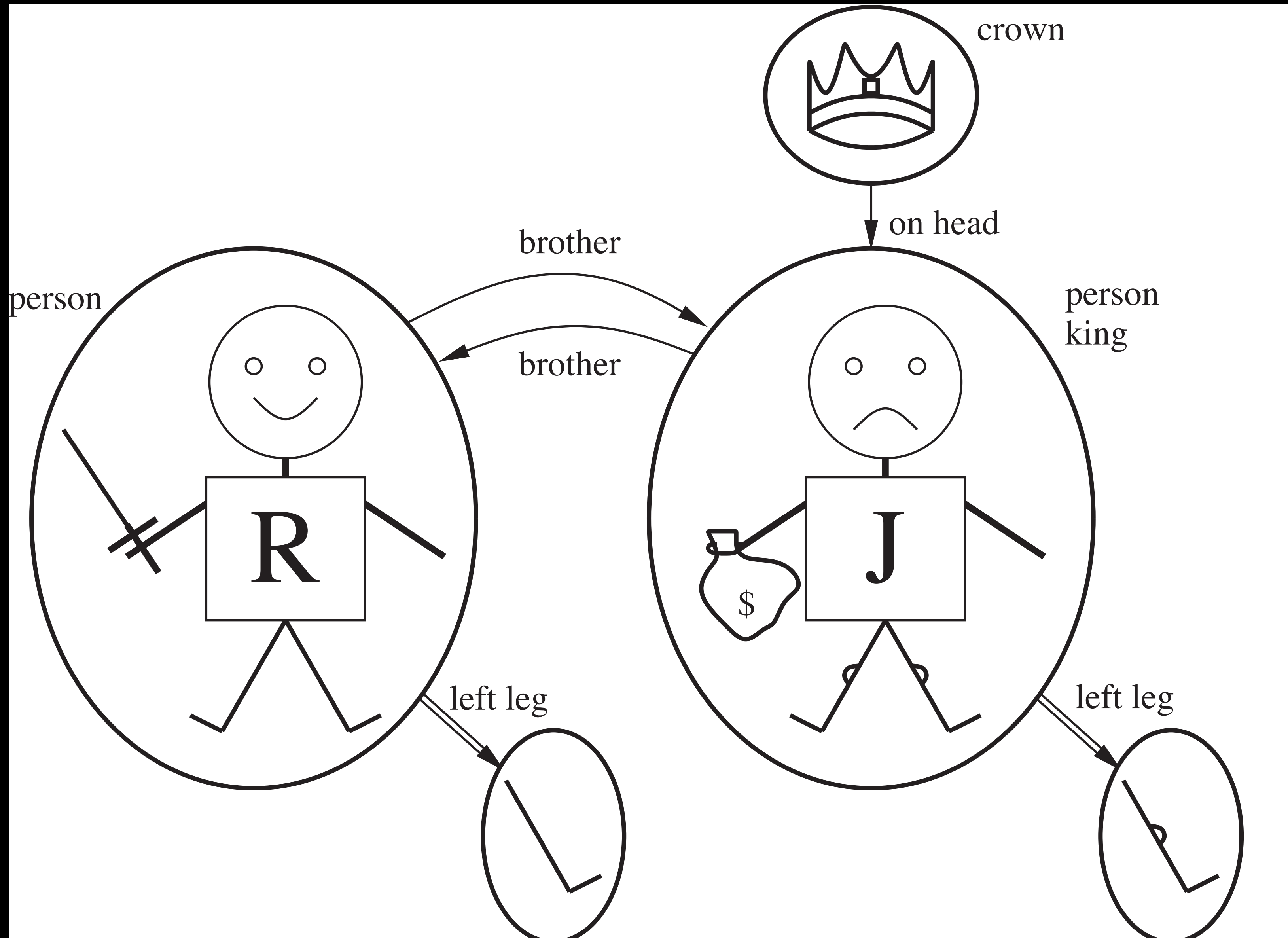
- Refer to relations between objects
- Relations = tuples (ordered sets) of objects that are related



$$Person = \{\langle R \rangle, \langle J \rangle\}$$

$$King = \{\langle J \rangle\}$$

$$Crown = \{\langle C \rangle\}$$



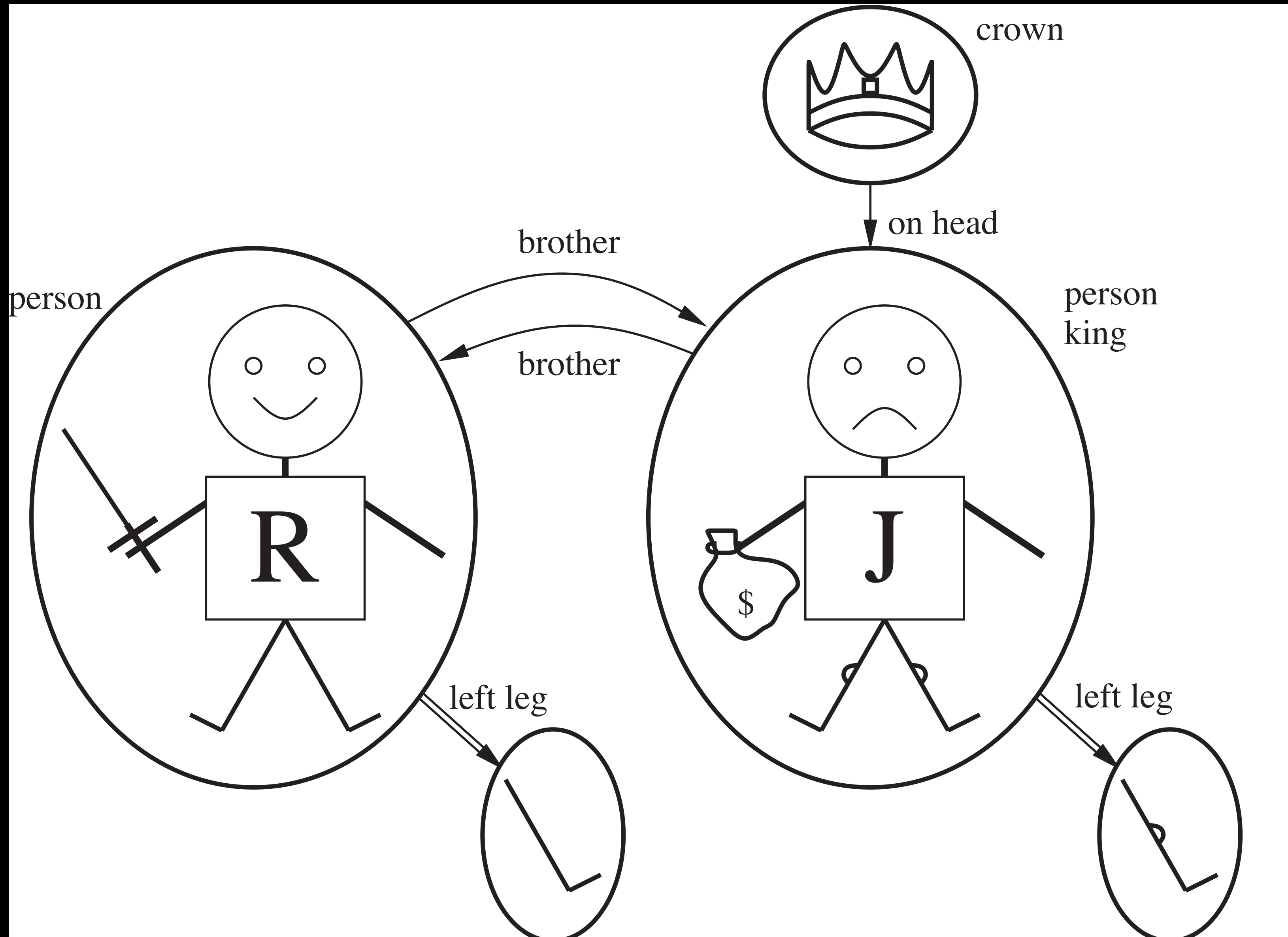
$$Brother = \{\langle R, J\rangle, \langle J, R\rangle\} \qquad OnHead = \{\langle C, J\rangle\}$$

Models for FOL: Relations

- Sets of tuples of objects
  - Unary: one object
  - Binary: pair of objects
  - Ternary: triple of objects
  - *n*-ary: *n*-tuple of objects

FOL: Functions

- Refer to objects in terms of other objects
- Functions = Total mapping from tuples of objects to objects



$$LeftLeg = \{\langle R\rangle \to LLR, \langle J\rangle \to LLJ\}$$

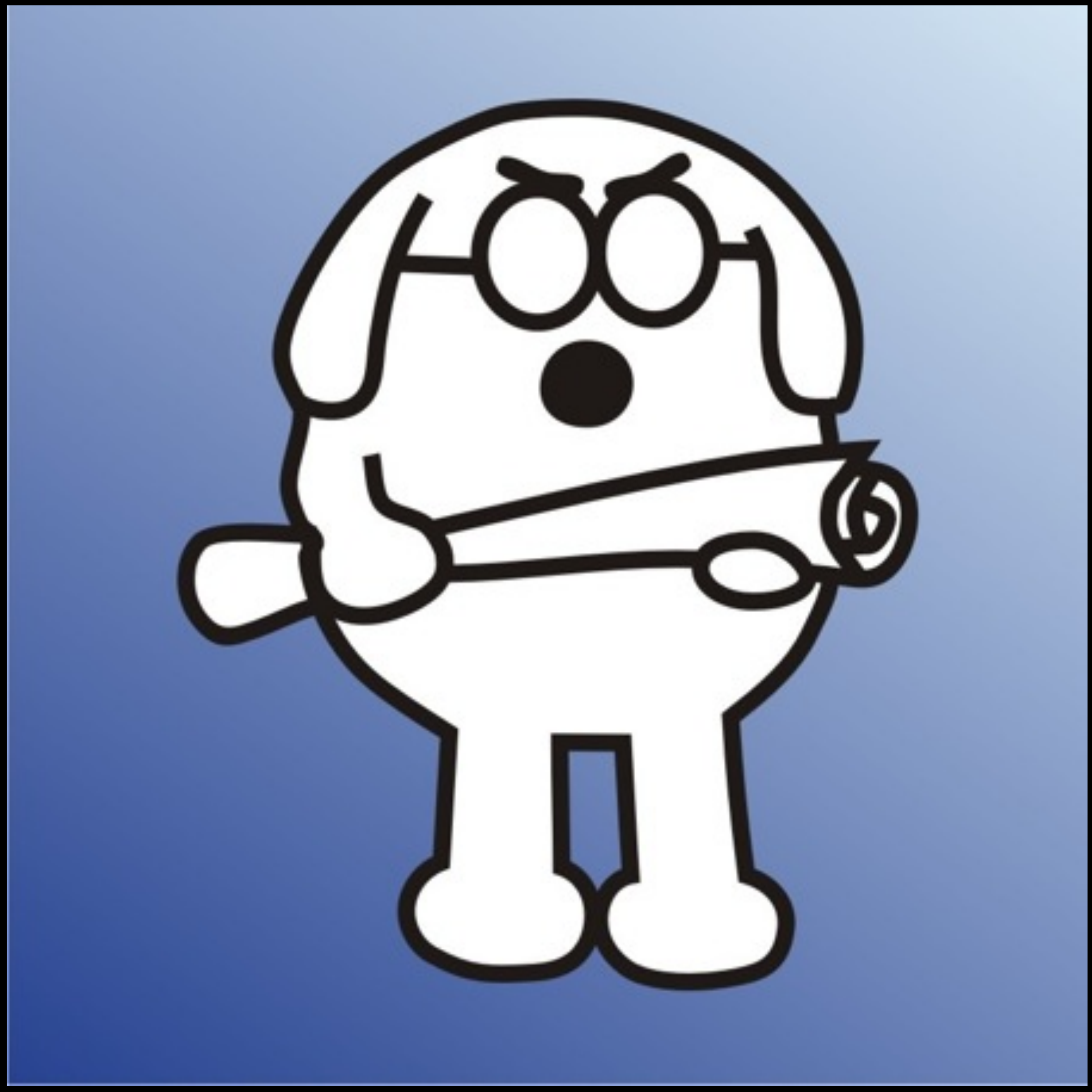
Models for FOL: Functions

- Total mappings from tuples of objects to objects
  - Unary: one object
  - Binary: pair of objects
  - Ternary: triple of objects
  - *n*-ary: *n*-tuple of objects

Models for FOL

- Domain: Non-empty set of objects in the world
- Relations: Sets of tuples of objects
- Functions: Total mappings from tuples of objects to objects

So What?



- Models are possible worlds
- We want to be able to compute whether a sentence is true in a model
- From that we can define entailment
- And then we can do inference ("follows from")!

Interpretation

- Mapping from elements a sentence to elements of a model
- Specifies exactly which objects, relations, and functions are referred to by the constant, predicate, and function symbols

Interpretation

- R refers to Richard the Lionheart
- J refers to evil King John
- $Brother$ refers to the brotherhood relation (that is, the set of tuples we saw before); similarly $OnHead$, $Person$, $King$, and $Crown$
- $LeftLeg$ refers to the "left leg of" function (that is, the mapping we saw before)

Models for FOL

- Domain: Non-empty set of objects in the world
- Interpretation that maps:
  - Constants to objects
  - Predicate symbols to relations between objects
  - Function symbols to total functions on objects

Atomic Sentences

- Predicate symbol + list of terms (arguments)
  - *Brother*(*R*, *J*)
  - *Married*(*Father*(*Richard*), *Mother*(*John*))
- True in a model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments

Interpretation

- R refers to Richard the Lionheart
- J refers to evil King John
- $Brother$ refers to the brotherhood relation (that is, the set of tuples we saw before); similarly $OnHead$, $Person$, $King$, and $Crown$
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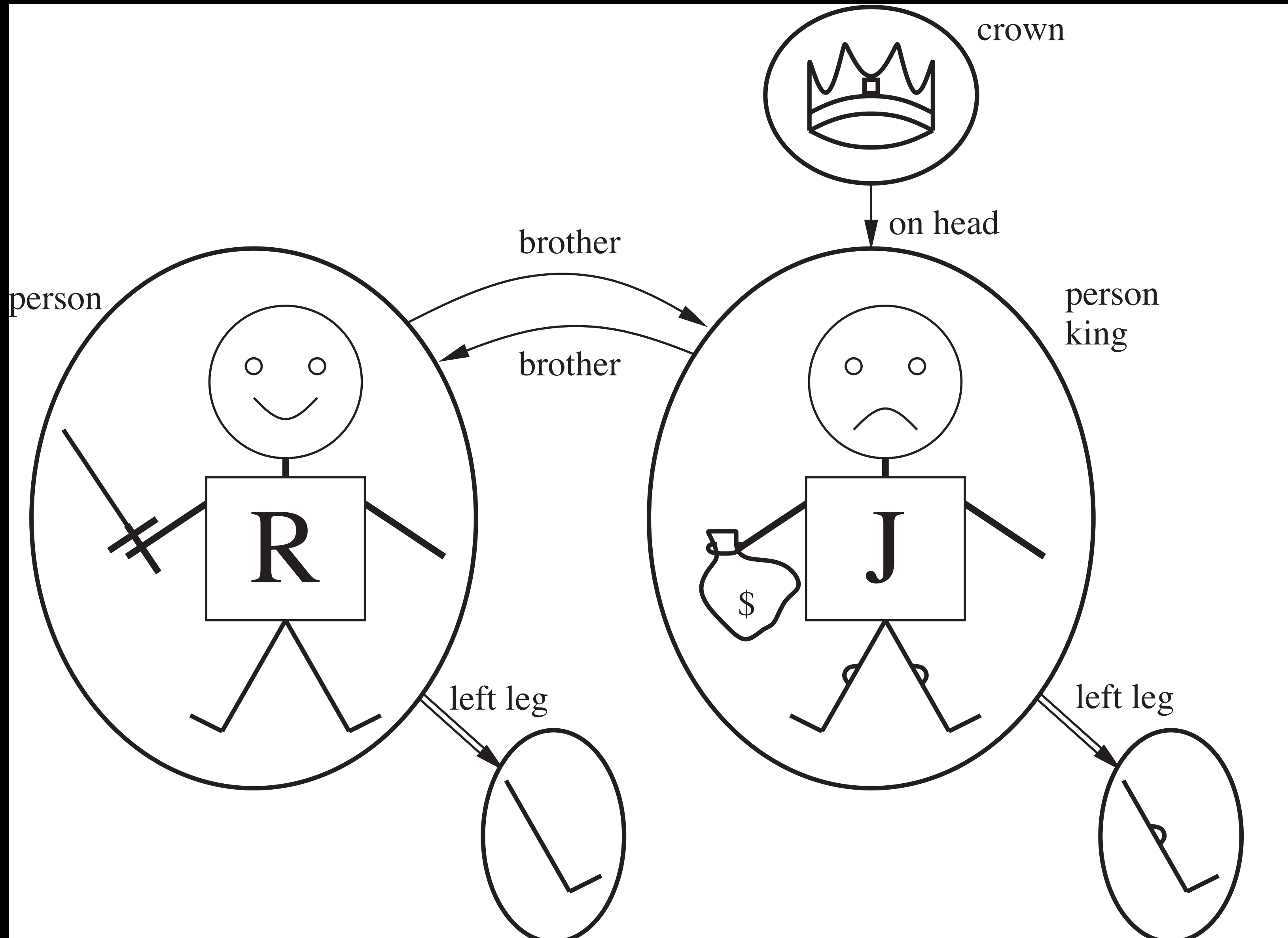
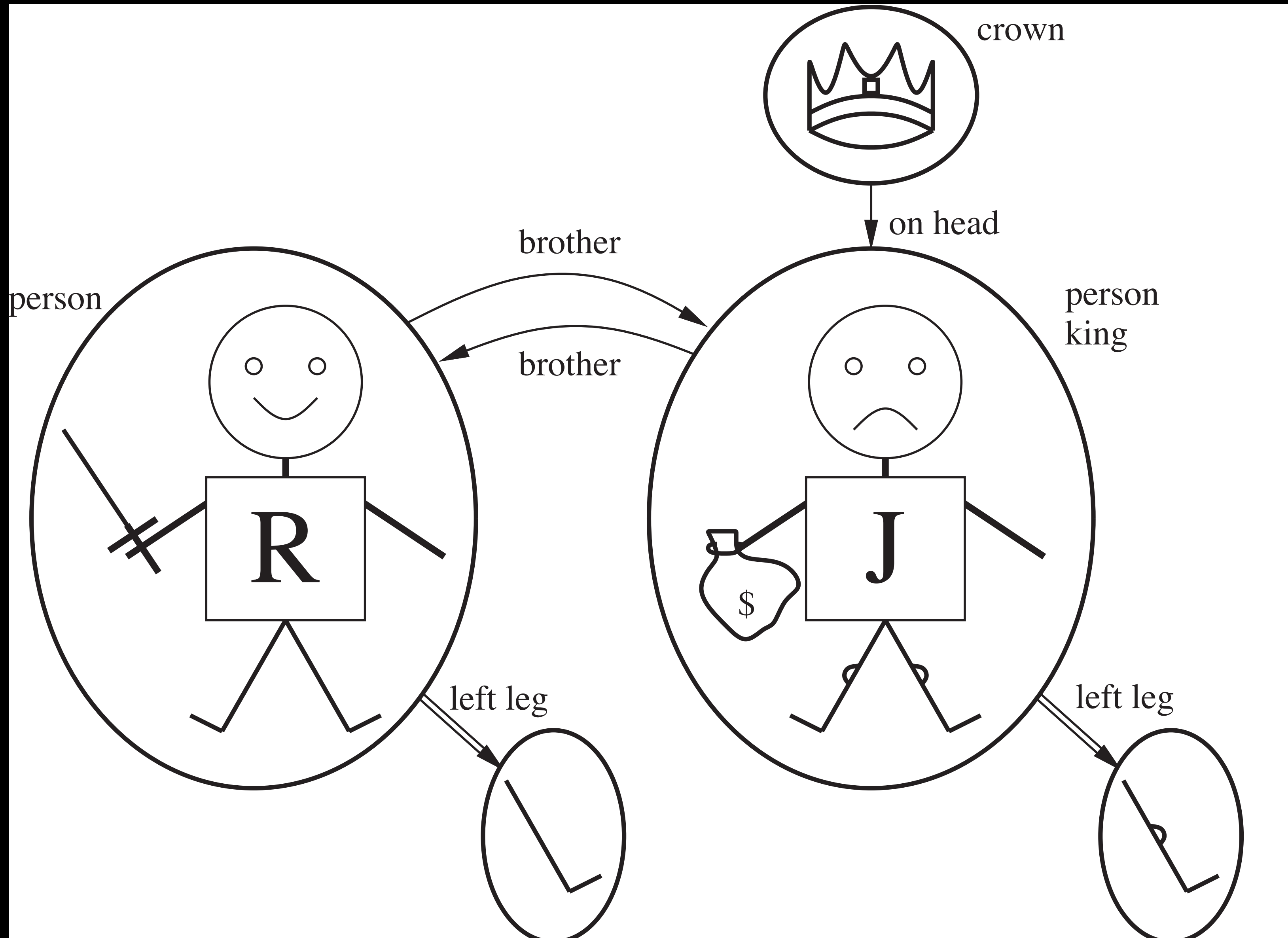


$$Brother = \{\langle R, J\rangle, \langle J, R\rangle\} \qquad OnHead = \{\langle C, J\rangle\}$$

Brother(R, J) ✓



$$Brother = \{\langle R, J\rangle, \langle J, R\rangle\} \qquad OnHead = \{\langle C, J\rangle\}$$

Brother(C, J) ✗

Atomic Sentences

- Predicate symbol + list of <u>terms</u> (arguments)
  - *Brother*(*R*, *J*)
  - *Married*(*Father*(*Richard*), *Mother*(*John*))
- True in a model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments

Complex Sentences

- Connectives combine sentences
  - $\neg Brother(LeftLeg(R), J)$
  - $Brother(R, J) \land Brother(J, R)$
  - $\neg King(R) \Rightarrow King(J)$
- Same semantics as propositional logic!
  - True in a model if truth values of arguments satisfies truth table for connective

First-Order Logic

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- Function symbols that denote a mapping from a tuple of objects to another object
- Predicate symbols that denote relations (sets of tuples of objects)
- Connectives

First-Order Logic

- Constants and variables that denote objects in the world
- Function symbols that denote a mapping from a tuple of objects to another object
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- Connectives
- Variables and Quantifiers

- “All dogs are mammals”
- “All dogs bark”
- “All people are mortal”
- “All rooms adjacent to pits are breezy”
- “Some room contains the wumpus”

Quantifiers

- If α is a sentence, then so are
  - $\forall x\ \alpha$ (universal quantifier)
  - $\exists x\ \alpha$ (existential quantifier)

Universal Quantifier

- $\forall x$ *P* says that *P* is true for <u>every</u> object *x*
- $\forall x$ *P* is true in a model if *P* is true in all possible <u>extended interpretations</u> of the interpretation in the model
- Each extended interpretation maps *x* to an object in the domain of the model

- $\forall x\ King(x) \Rightarrow Person(x)$
- Intended interpretation:
  - Domain: Richard, John, crown, Richard's left leg, John's left leg
- Extended interpretations:
  - x → Richard
  - x → John
  - x → the crown
  - x → Richard's left leg
  - x → John's left leg

- $\forall x$ King(x) ⇒ Person(x)
- Extended interpretations:
  - Richard is a king ⇒ Richard is a person
  - John is a king ⇒ John is a person
  - The crown is a king ⇒ The crown is a person
  - Richard's left leg is a king ⇒ Richard's left leg is a person
  - Richard's left leg is a king ⇒ Richard's left leg is a person

Universal Quantifier

- $\forall x\ P$ says that P is true for <u>every</u> object x
- $\forall x\ P$ is true in a model if P is true in all possible <u>extended interpretations</u> of the interpretation in the model
- Each extended interpretation maps x to an object in the domain of the model

Existential Quantifier

- $\exists x\ P$ says that *P* is true for some object *x*
- $\exists x\ P$ is true in a model if *P* is true in at least one possible extended interpretation of the interpretation in the model
- Each extended interpretation maps *x* to an object in the domain of the model

- $\exists x\ Crown(x) \land OnHead(x,J)$
- Intended interpretation:
  - Domain: Richard, John, crown, Richard's left leg, John's left leg
- Extended interpretations:
  - $x \rightarrow$ Richard
  - $x \rightarrow$ John
  - $x \rightarrow$ the crown
  - $x \rightarrow$ Richard's left leg
  - $x \rightarrow$ John's left leg

- $\exists x$ Crown(x) $\wedge$ OnHead(x,J)
- Extended interpretations:
  - Richard is a crown $\wedge$ Richard is on John's headon John's head
  - John is a crown $\wedge$ John is on John's head
  - The crown is a crown $\wedge$ The crown is on John's head
  - Richard's left leg is a crown $\wedge$ Richard's left leg is on John's head
  - Richard's left leg is a crown $\wedge$ Richard's left leg is on John's head

Existential Quantifier

- $\exists x\ P$ says that P is true for <u>some</u> object x
- $\exists x\ P$ is true in a model if P is true in at least one possible <u>extended interpretation</u> of the interpretation in the model
- Each extended interpretation maps x to an object in the domain of the model

Nested Quantifiers

- "Brothers are siblings"
  - $\forall x \; \forall y \; Brother(x,y) \Rightarrow Sibling(x,y)$
- "Everybody loves somebody"
  - $\forall x \; \exists y \; Loves(x,y)$
- "Somebody is loved by everybody"
  - $\exists y \; \forall x \; Loves(x,y)$

Models for FOL

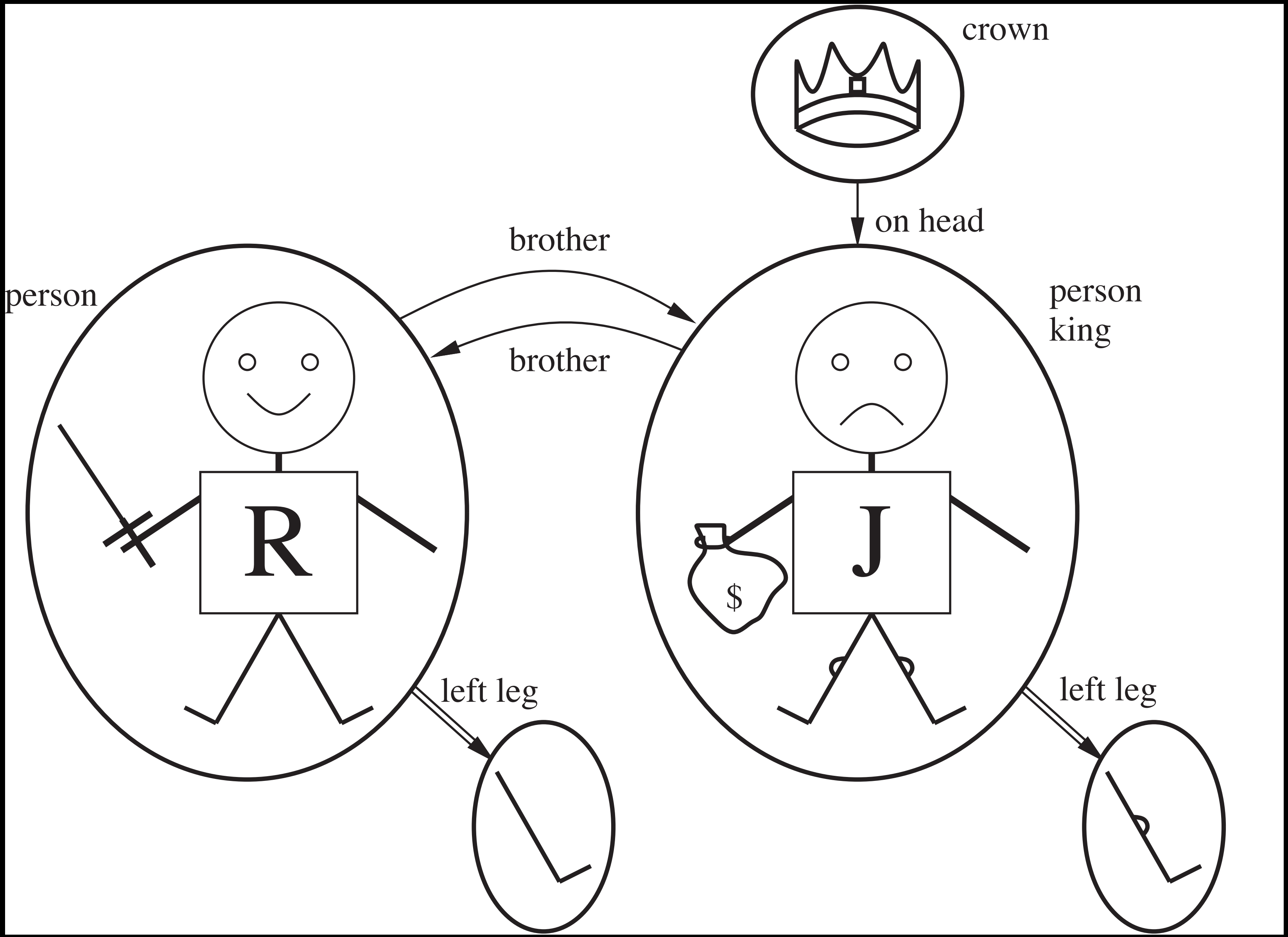
- Domain: Non-empty set of objects in the world
- Interpretation that maps:
  - Constants to objects
  - Predicate symbols to relations between objects
  - Function symbols to total functions on objects

Entailment

- α entails β when:
  - β is true in **every** world considered possible by α
  - Every model of α is also a model of β
  - *Models(*α*)* ⊆ *Models(*β*)*

All Possible Models

- # of objects from 1 to ∞
- Some constants refer to the same object
- Some objects are not referred to by any constant (“unnamed”)
- Relations and functions defined over sets of subsets of objects
- Variables range over all possible objects in extended interpretations



137,506,194,466 models with ≤6 objects!

Computing Entailment

- Number of models (probably) unbounded
  - And anyway hard to evaluate truth in a model
- Can't do model checking
- Look for inference rules, do theorem proving

For Next Time
AIMA Ch. 9