CSC242: Intro to AI

Lecture 19:
Neural Networks Part II

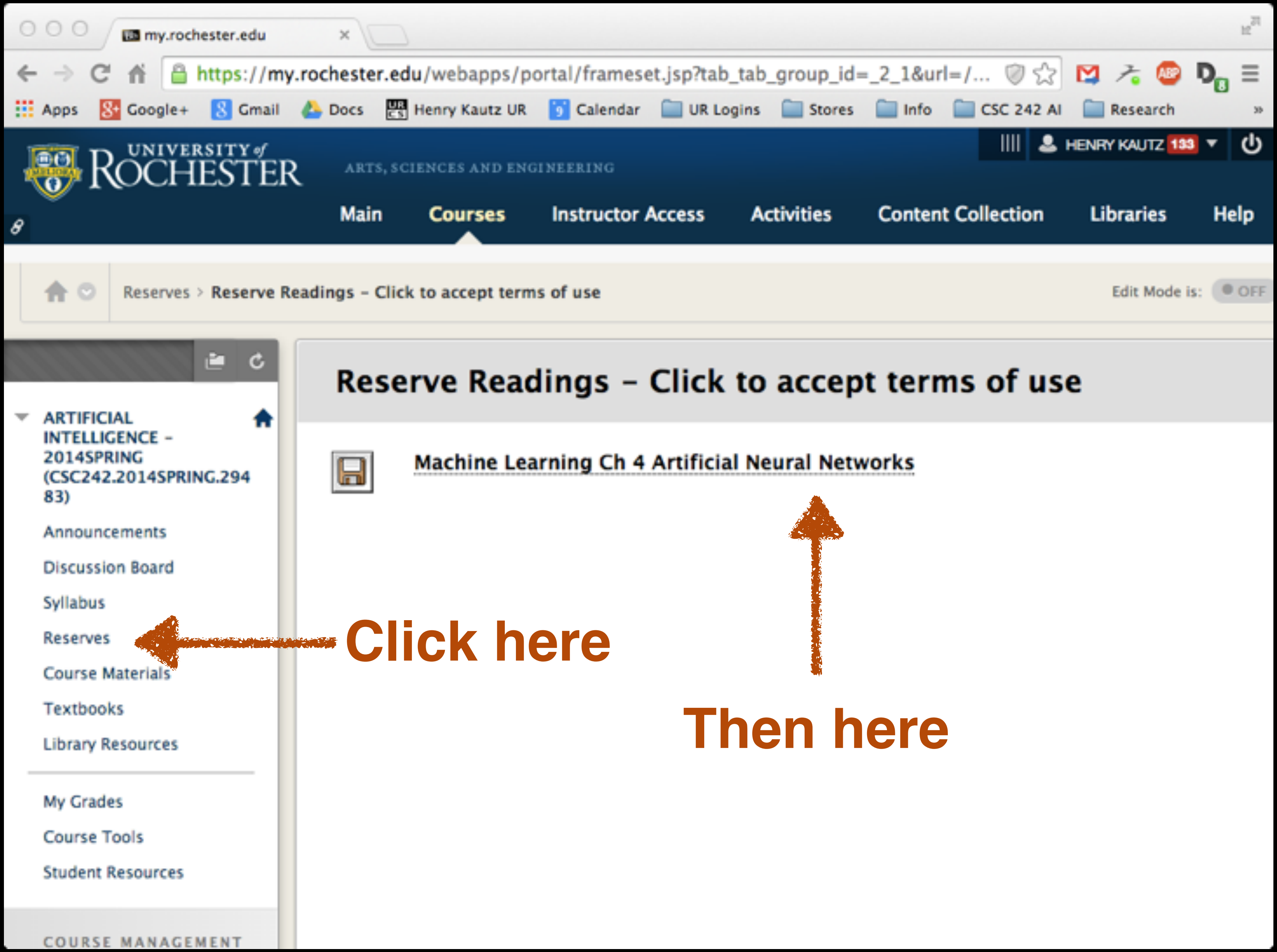
Part I Review

Reserve Readings

Artificial Neural Networks

Chapter 4 of *Machine Learning*

by Tom Mitchell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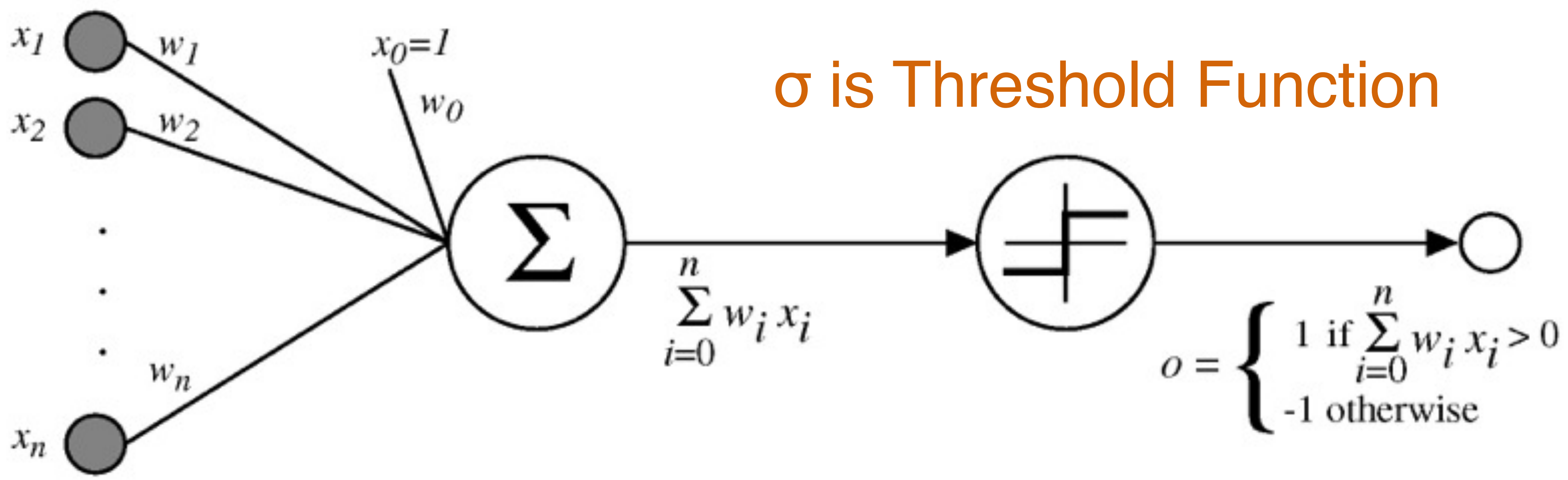
Artificial Neural Networks

[Read Ch. 4]

[Recommended exercises 4.1, 4.2, 4.5, 4.9, 4.11]

- Threshold units
- Gradient descent
- Multilayer networks
- Backpropagation
- Hidden layer representations
- Example: Face Recognition
- Advanced topics

Perceptron



x_1
x_2
x_n
w_1
w_2
w_n
$x_0=1$
w_0
Σ
$\sum_{i=0}^{n} w_i x_i$
$o = \begin{cases} 1 & \text{if } \sum_{i=0}^{n} w_i x_i > 0 \\ -1 & \text{otherwise} \end{cases}$

σ is Threshold Function

$$o(x_1, \ldots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Sometimes we'll use simpler vector notation:

$$o(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Linear Classifier

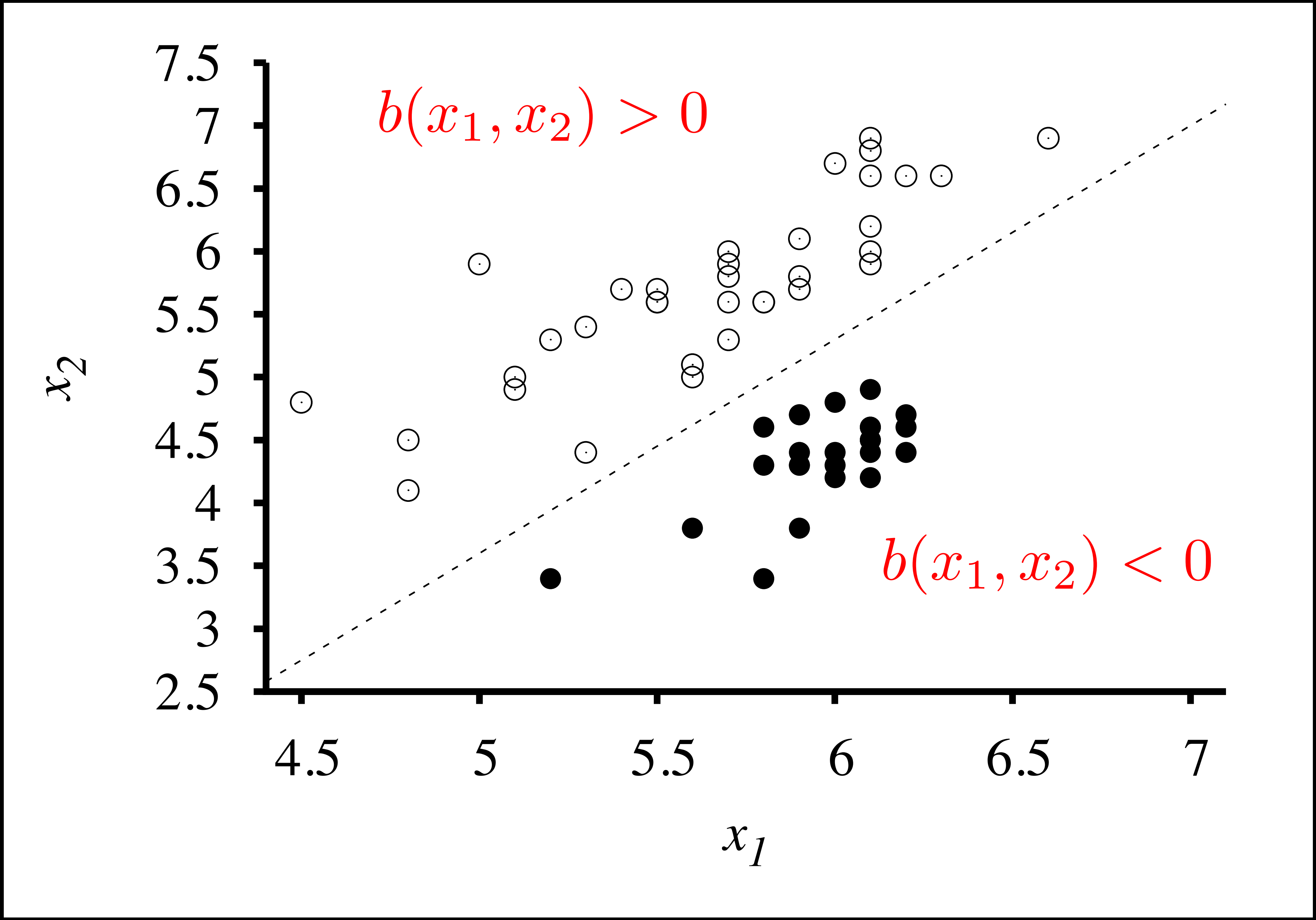
$$w_0 + w_1 x_1 + w_2 x_2 = 0$$

$$\mathbf{w} \cdot \mathbf{x} = 0$$

All instances of one class are above the line: $\mathbf{w} \cdot \mathbf{x} > 0$

All instances of one class are below the line: $\mathbf{w} \cdot \mathbf{x} < 0$

$$h_{\mathbf{w}}(\mathbf{x}) = Threshold(\mathbf{w} \cdot \mathbf{x})$$



$$b(x_1, x_2) = x_2 - 1.7x_1 + 4.9$$

Training

- Training is using data to set the weights for a perceptron (or network of perceptrons)
- Idea:
  - Start with random weights
  - For each piece of data:
    - Set inputs to the data features
    - Compare output to the label (target value)
    - If not same then adjust the weights

Perceptron training rule

$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

Where:

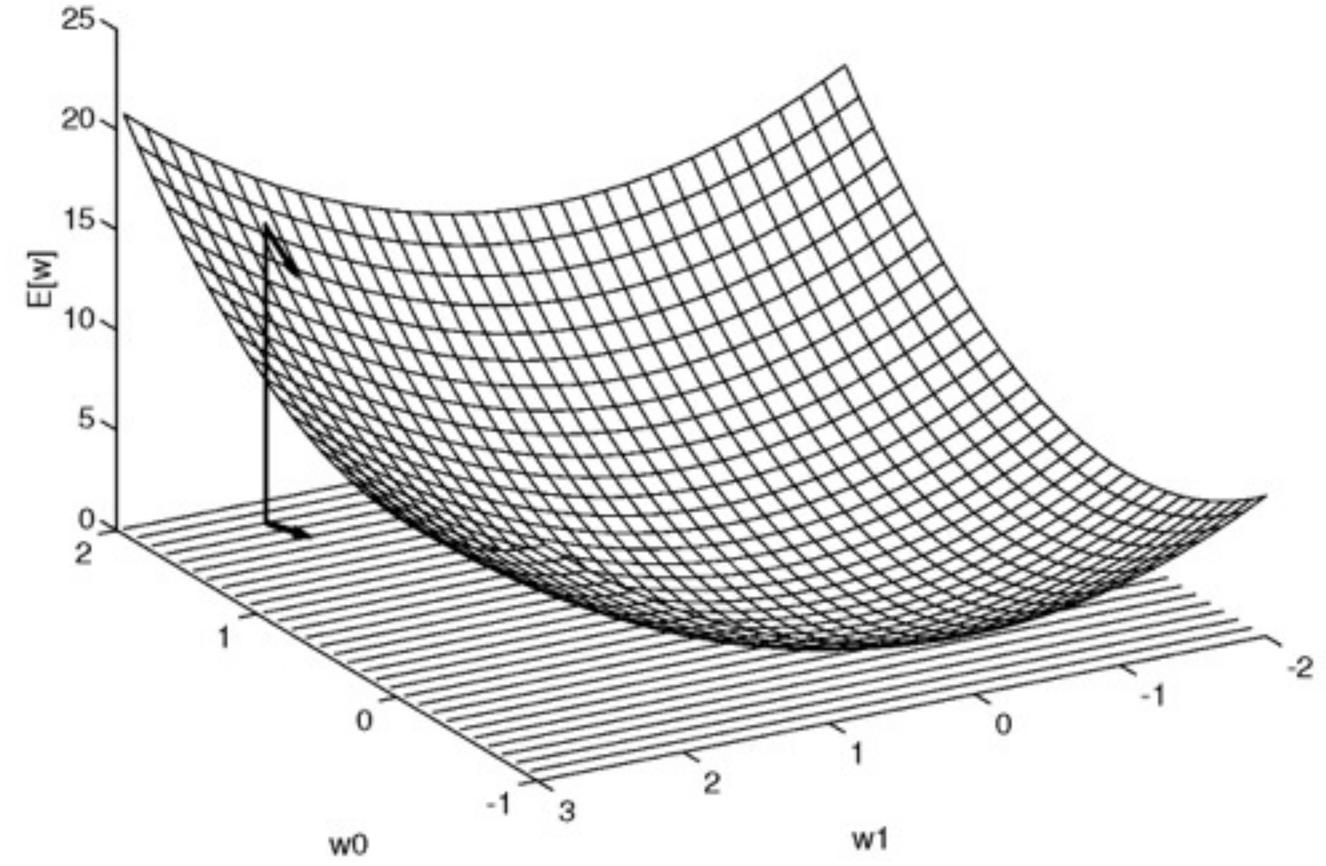
- $t = c(\vec{x})$ is target value
- o is perceptron output
- η is small constant (e.g., .1) called *learning rate*

Gradient Descent

```
GRADIENT-DESCENT($training\_examples, \eta$)

    Each training example is a pair of the form
    $\langle \vec{x}, t \rangle$, where $\vec{x}$ is the vector of input values,
    and $t$ is the target output value. $\eta$ is the
    learning rate (e.g., .05).

  • Initialize each $w_i$ to some small random value
  • Until the termination condition is met, Do
      – Initialize each $\Delta w_i$ to zero.
      – For each $\langle \vec{x}, t \rangle$ in $training\_examples$, Do
          ∗ Input the instance $\vec{x}$ to the unit and
            compute the output $o$
          ∗ For each linear unit weight $w_i$, Do
                $\Delta w_i \leftarrow \Delta w_i + \eta(t - o)x_i$
      – For each linear unit weight $w_i$, Do
                $w_i \leftarrow w_i + \Delta w_i$
```



Gradient

$$\nabla E[\vec{w}] \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \cdots \frac{\partial E}{\partial w_n}\right]$$

Training rule:

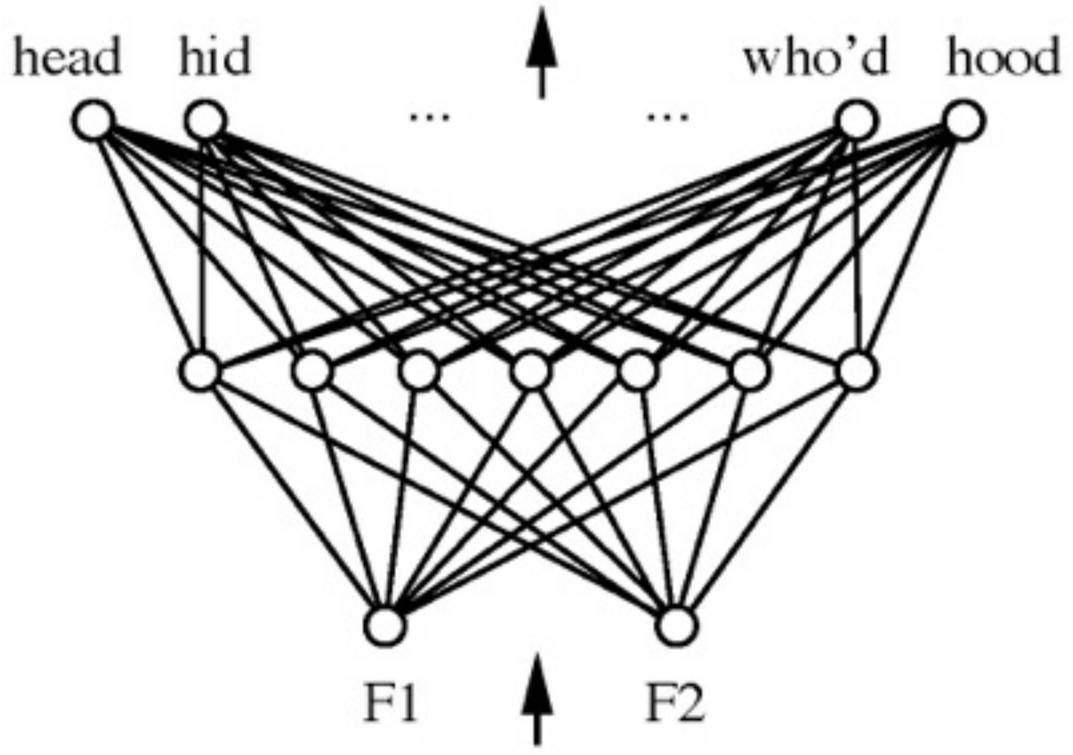
$$\Delta \vec{w} = -\eta \nabla E[\vec{w}]$$

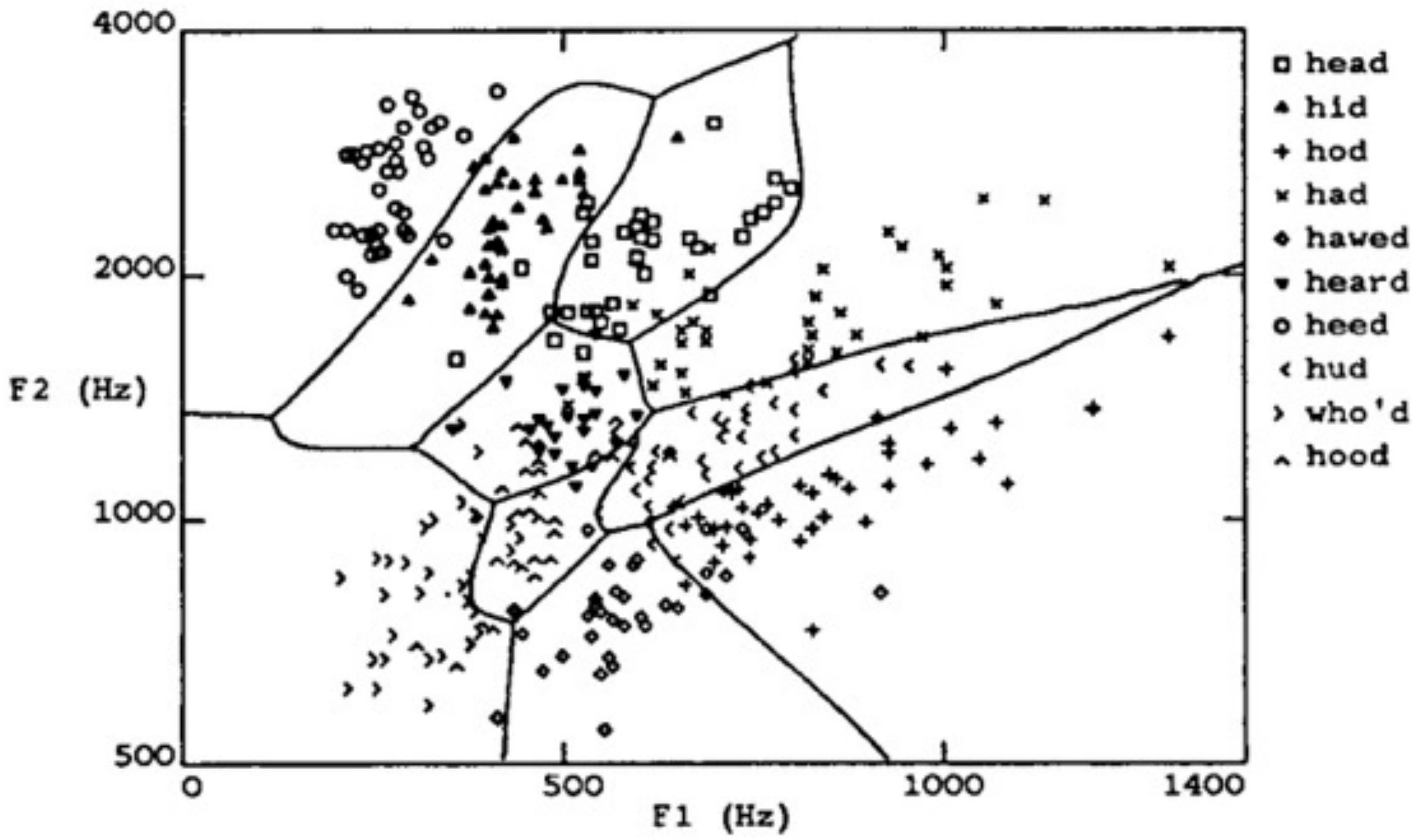
i.e.,

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

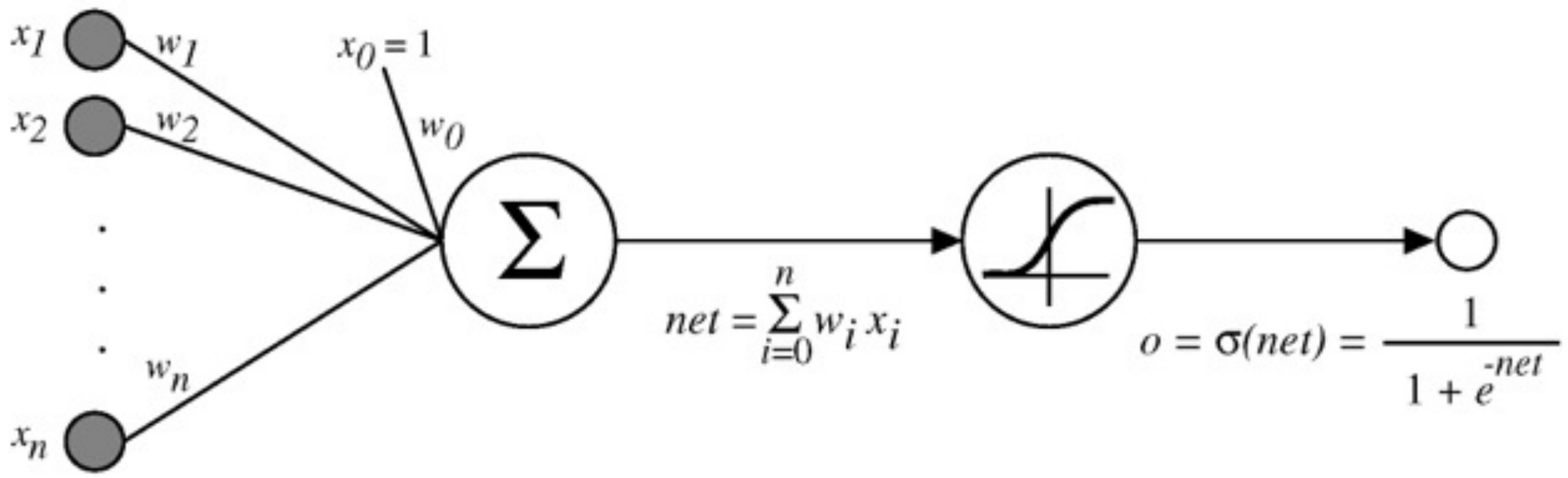
Part II
Multilayer Neural Nets
Backpropagation
Deep Learning
Face Recognition Project

Multilayer Networks of Sigmoid Units





Sigmoid Unit



x_1
w_1
x_2
w_2
$x_0 = 1$
w_0
w_n
x_n
Σ
$net = \sum_{i=0}^{n} w_i x_i$
$o = \sigma(net) = \frac{1}{1 + e^{-net}}$

How does this differ from a Perceptron?

Smooth threshold

Range 0 to 1, not -1 to +1

$\sigma(x)$ is the sigmoid function

$$\frac{1}{1 + e^{-x}}$$

Nice property: $\frac{d\sigma(x)}{dx} = \sigma(x)(1 - \sigma(x))$

We can derive gradient decent rules to train

- One sigmoid unit
- *Multilayer networks* of sigmoid units $\rightarrow$ Backpropagation

Error Gradient for a Sigmoid Unit

Error Gradient for a Sigmoid Unit

$$\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d 2(t_d - o_d) \ \frac{\partial}{\partial w_i} (t_d - o_d) \\
&= \sum_d (t_d - o_d) \left(- \frac{\partial o_d}{\partial w_i} \right) \\
&= - \sum_d (t_d - o_d) \ \frac{\partial o_d}{\partial net_d} \ \frac{\partial net_d}{\partial w_i}
\end{aligned}$$

But we know:

$$\frac{\partial o_d}{\partial net_d} = \frac{\partial \sigma(net_d)}{\partial net_d} = o_d(1 - o_d)$$

$$\frac{\partial net_d}{\partial w_i} = \frac{\partial (\vec{w} \cdot \vec{x}_d)}{\partial w_i} = x_{i,d}$$

So:

$$\frac{\partial E}{\partial w_i} = - \sum_{d \in D} (t_d - o_d) o_d (1 - o_d) x_{i,d}$$

Compare

- Perceptron:

$$\frac{\partial E}{\partial w_i} = \sum_d (t_d - o_d)(-x_{i,d})$$

$$\Delta w_i = \eta(t - o)x_i$$

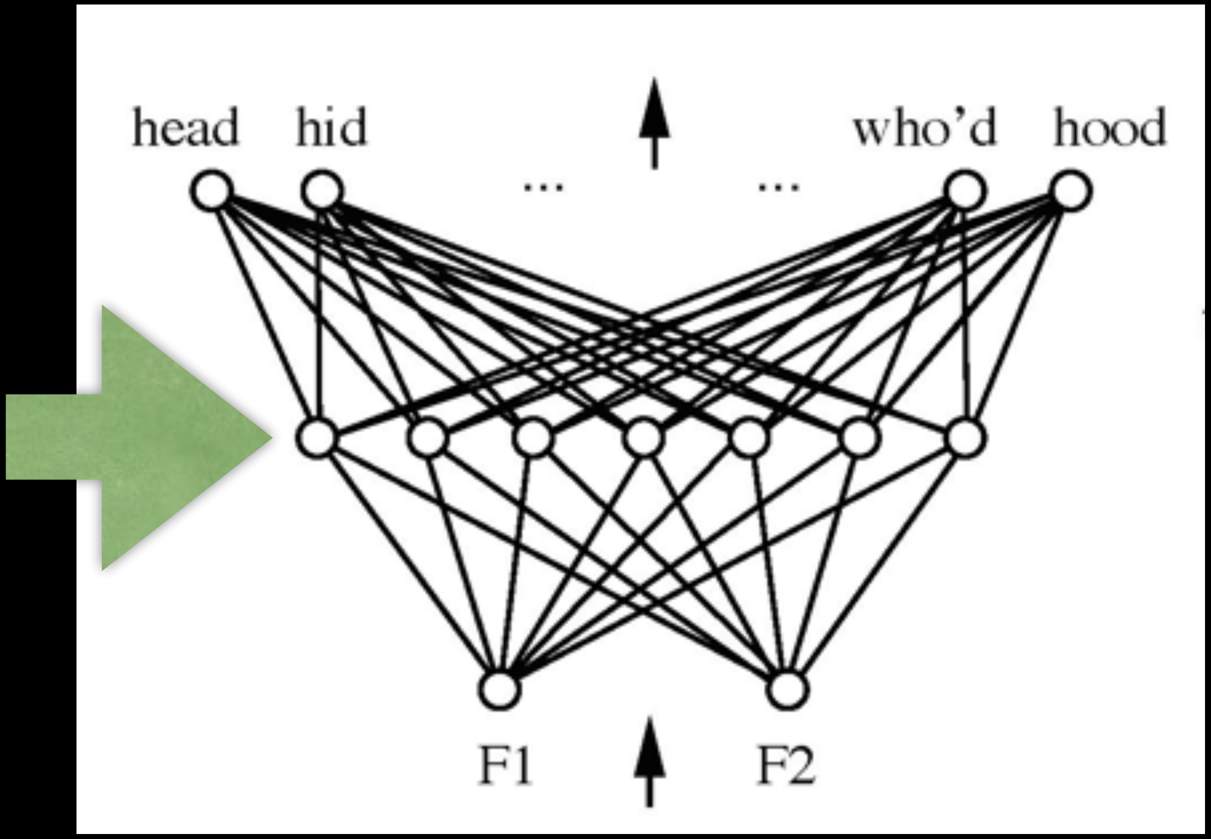
- Sigmoidal Unit:

$$\frac{\partial E}{\partial w_i} = -\sum_{d \in D} (t_d - o_d) o_d (1 - o_d) x_{i,d}$$

$$\Delta w_i = \eta(t - o)o(1 - o)x_i$$

Multilayer Networks

- How to train the hidden (middle) units?
  - We don't have a target output for them!
- Idea:
  - First, calculate deltas for output units
  - Use the weighted average of these deltas to compute a delta for each hidden unit



Backpropagation Algorithm

```
Initialize all weights to small random numbers.
Until satisfied, Do
  • For each training example, Do
      1. Input the training example to the network
         and compute the network outputs
      2. For each output unit k
             $\delta_k \leftarrow o_k(1 - o_k)(t_k - o_k)$
      3. For each hidden unit h
             $\delta_h \leftarrow o_h(1 - o_h) \sum_{k \in outputs} w_{h,k}\delta_k$
      4. Update each network weight $w_{i,j}$
             $w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$
         where
             $\Delta w_{i,j} = \eta \delta_j x_{i,j}$
```

Explain each piece of this equation!

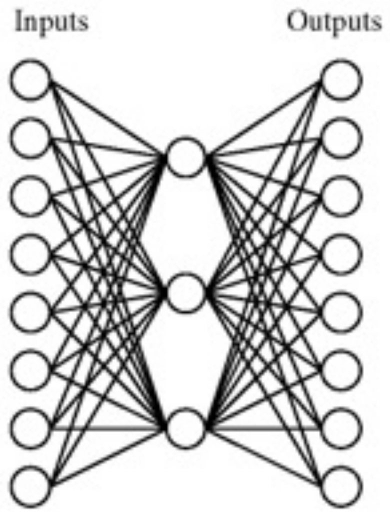
More on Backpropagation

- Gradient descent over entire *network* weight vector
- Easily generalized to arbitrary directed graphs
- Will find a local, not necessarily global error minimum
  - In practice, often works well (can run multiple times)
- Often include weight *momentum* α

$$\Delta w_{i,j}(n) = \eta \delta_j x_{i,j} + \alpha \Delta w_{i,j}(n-1)$$

- Minimizes error over *training* examples
  - Will it generalize well to subsequent examples?
- Training can take thousands of iterations $\rightarrow$ slow!
- Using network after training is very fast

Learning Hidden Layer Representations



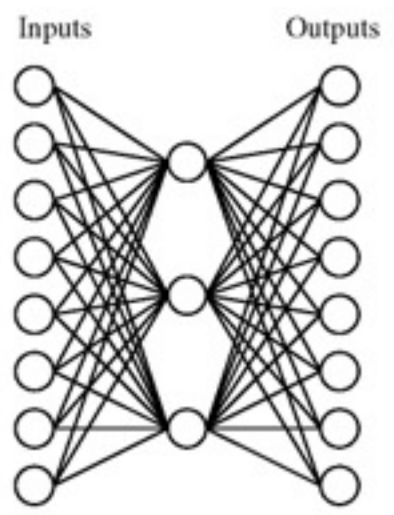
A target function:

Input		Output
10000000	$\rightarrow$	10000000
01000000	$\rightarrow$	01000000
00100000	$\rightarrow$	00100000
00010000	$\rightarrow$	00010000
00001000	$\rightarrow$	00001000
00000100	$\rightarrow$	00000100
00000010	$\rightarrow$	00000010
00000001	$\rightarrow$	00000001

Can this be learned??

Learning Hidden Layer Representations

A network:

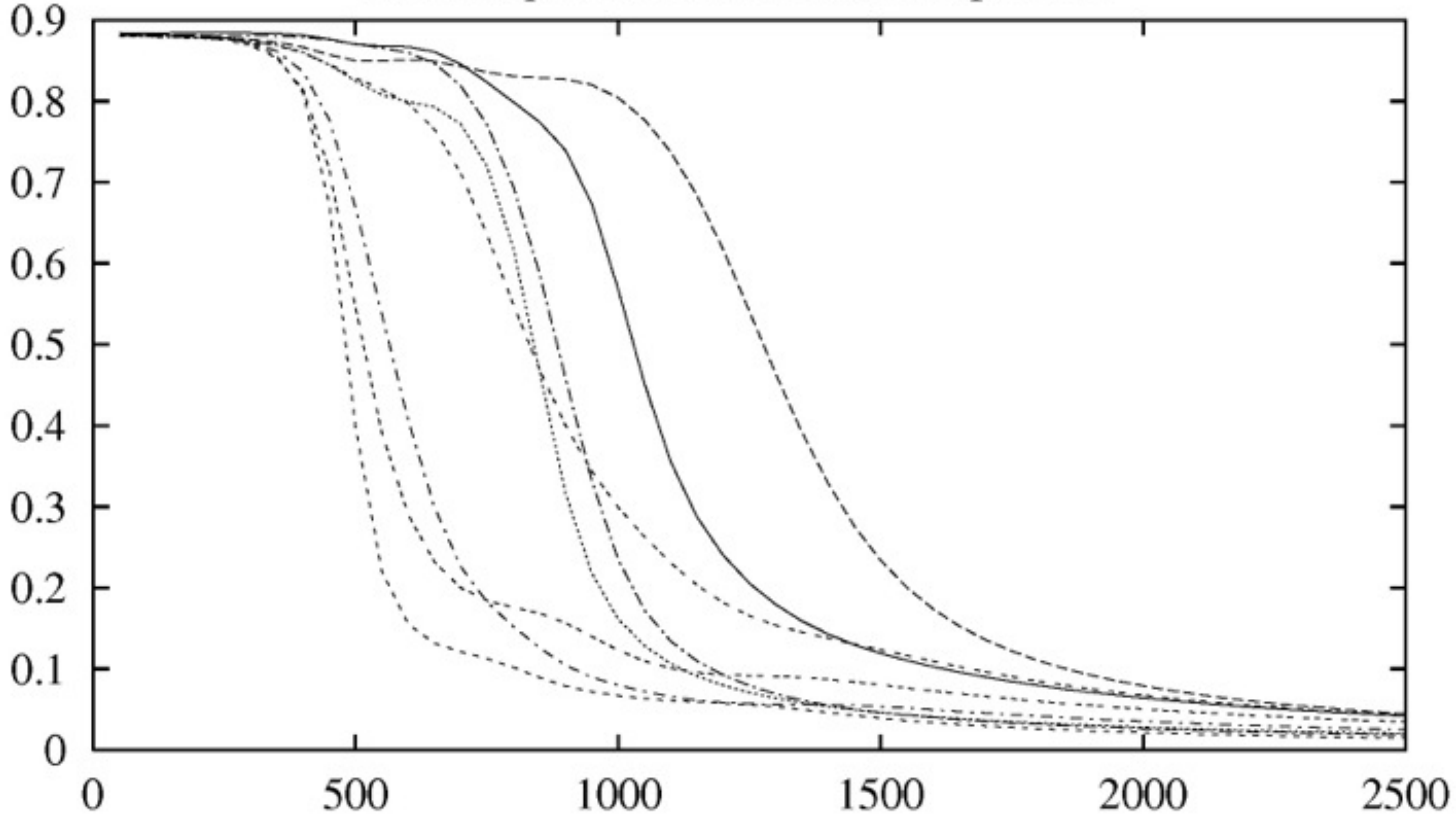


Learned hidden layer representation:

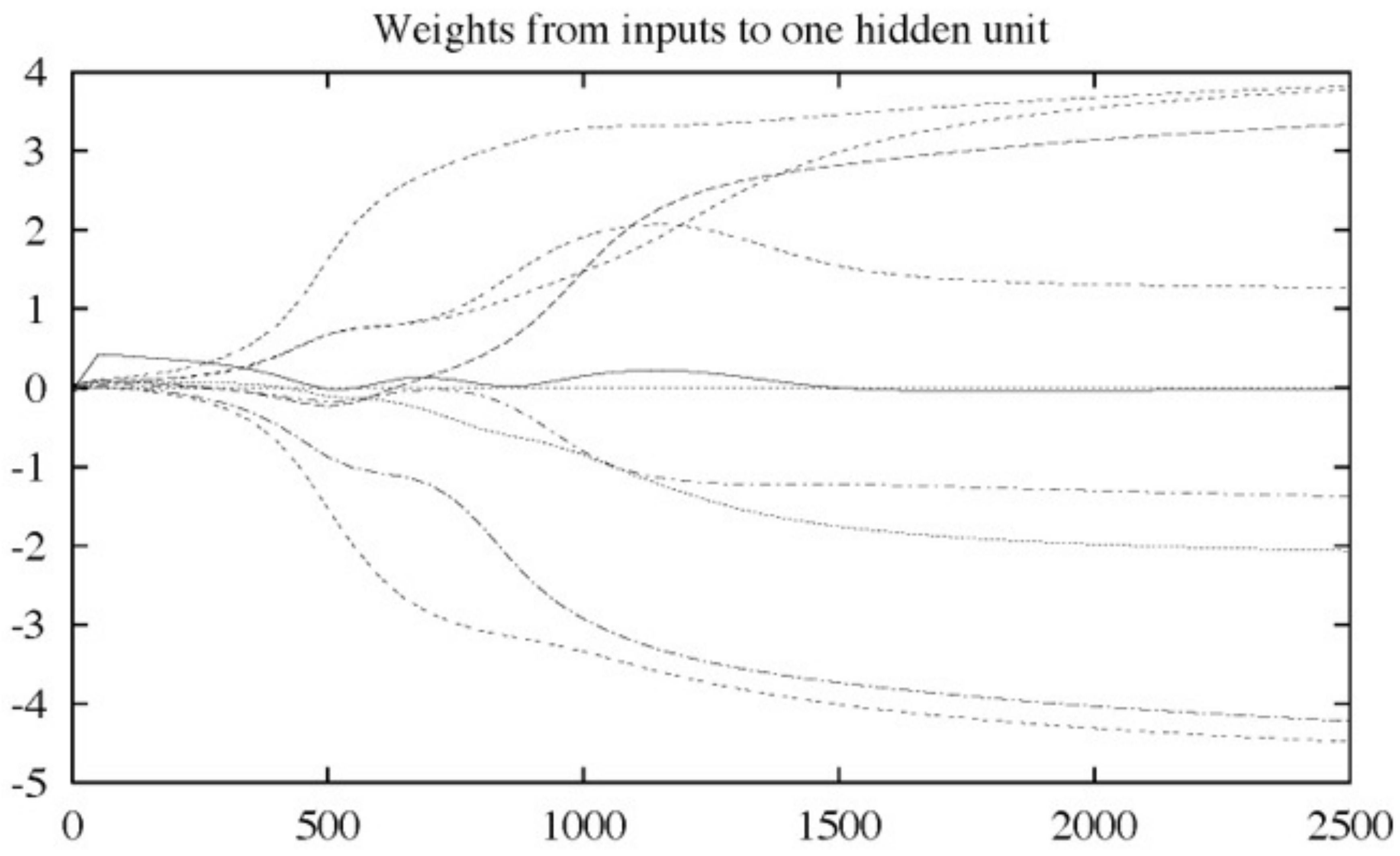
Input		Hidden Values				Output
10000000	→	.89	.04	.08	→	10000000
01000000	→	.01	.11	.88	→	01000000
00100000	→	.01	.97	.27	→	00100000
00010000	→	.99	.97	.71	→	00010000
00001000	→	.03	.05	.02	→	00001000
00000100	→	.22	.99	.99	→	00000100
00000010	→	.80	.01	.98	→	00000010
00000001	→	.60	.94	.01	→	00000001

Training

Sum of squared errors for each output unit



Training


Weights from inputs to one hidden unit
4
3
2
1
0
-1
-2
-3
-4
-5
0
500
1000
1500
2000
2500

Convergence of Backpropagation

Gradient descent to some local minimum

- Perhaps not global minimum...
- Add momentum
- Stochastic gradient descent
- Train multiple nets with different inital weights

Nature of convergence

- Initialize weights near zero
- Therefore, initial networks near-linear
- Increasingly non-linear functions possible as training progresses

Expressive Capabilities of ANNs

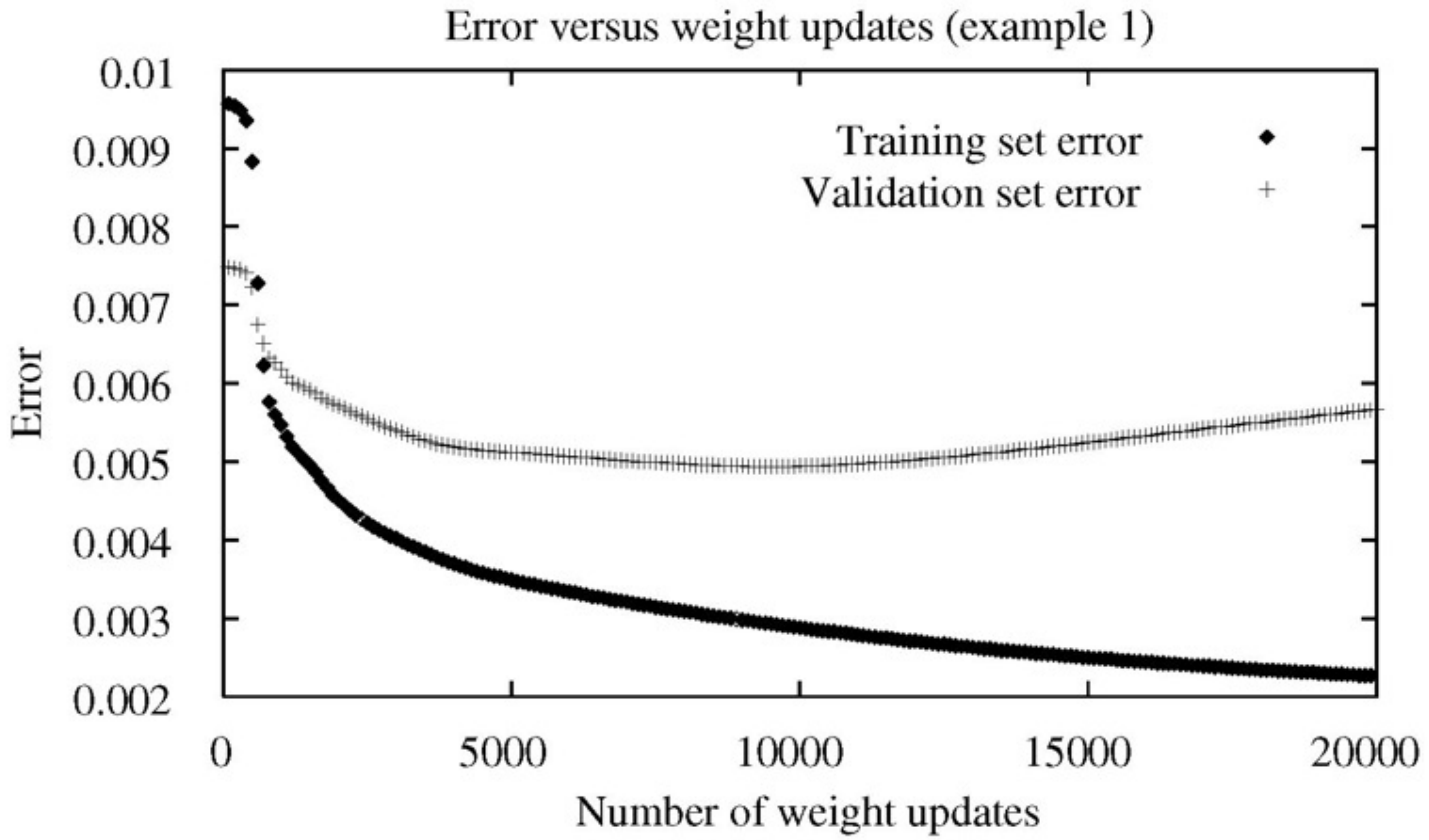
Boolean functions:

- Every boolean function can be represented by network with single hidden layer
- but might require exponential (in number of inputs) hidden units

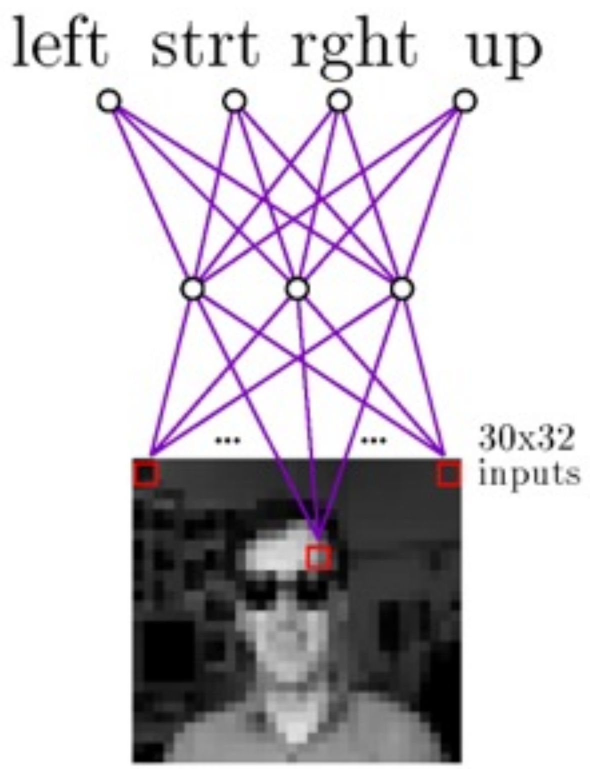
Continuous functions:

- Every bounded continuous function can be approximated with arbitrarily small error, by network with one hidden layer [Cybenko 1989; Hornik et al. 1989]
- Any function can be approximated to arbitrary accuracy by a network with two hidden layers [Cybenko 1988].

Using Validation Set to Prevent Overfitting



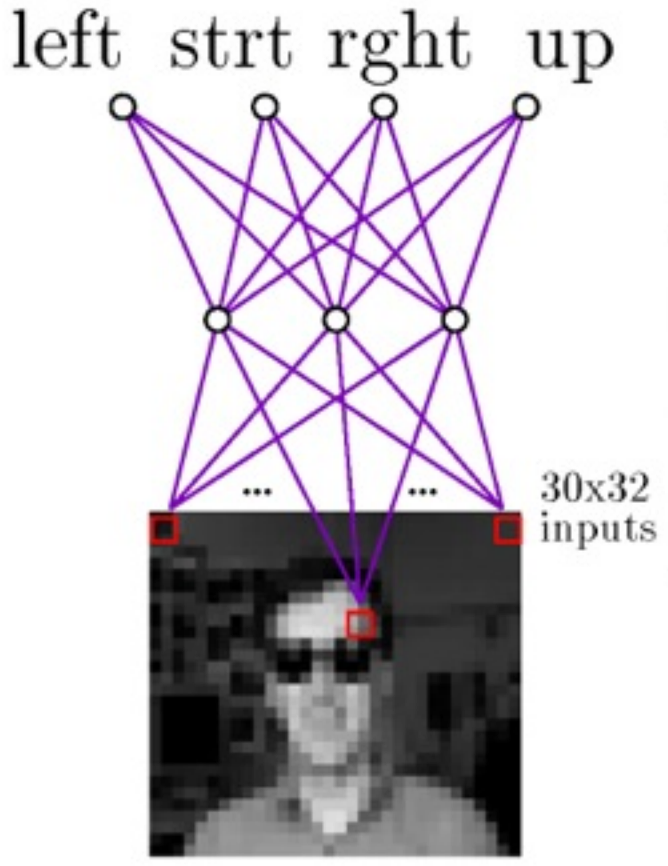
Neural Nets for Face Recognition



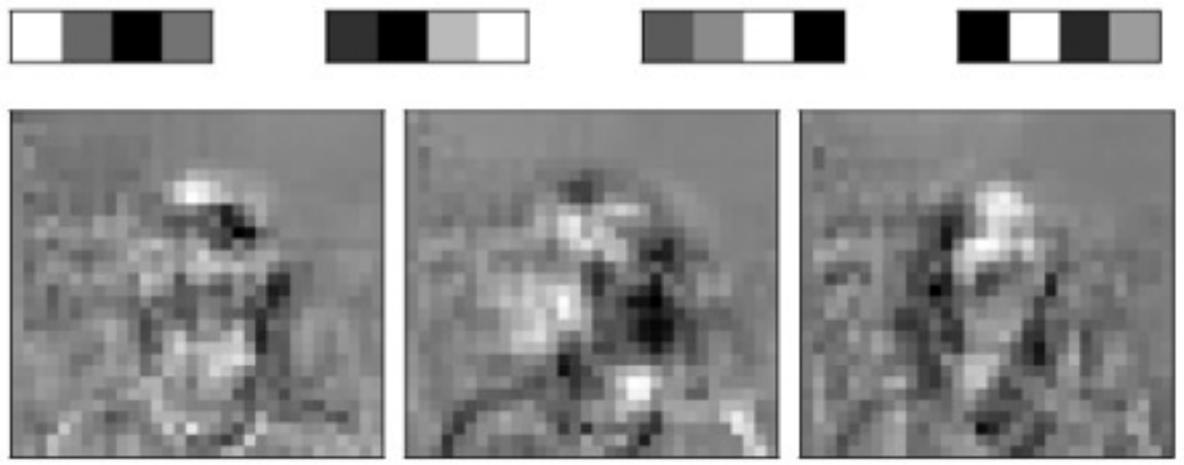
Typical input images

90% accurate learning head pose, and recognizing 1-of-20 faces

Learned Hidden Unit Weights



Learned Weights



Typical input images

The code for this assignment is broken into several modules:

- `facetrain.c`: the top-level program which uses all of the modules below to implement an image classification system. You will need to modify this code to change network sizes and learning parameters. The performance evaluation routines `performance_on_imagelist()` and `evaluate_performance()` are also in this module; you will need to modify these for your face and expression recognizers.

- `imagenet.c`: interface routines for loading images into the input units of a network, and setting up target vectors for training. You will need to modify the routine `load_target`, when implementing the face recognizer and the pose recognizer, to set up appropriate target vectors for the output encodings you choose.

- `backprop.c`, `backprop.h`: the neural network package. Supports three-layer fully-connected feedforward networks, using the backpropagation algorithm for weight tuning. Provides high level routines for creating, training, and using networks. **You will not need to modify any code in this module to complete the assignment.**

Coming Up

- April 17 - In-Class Workshop for Project 3
  - Live highly attractive TAs will personally help you complete the project!
  - An afternoon you will not soon forget!