Everything You Need to Know

(since the midterm)

Diagnosis

- Abductive diagnosis: a minimal set of (positive and negative) assumptions that entails the observations
- Consistency-based diagnosis: a minimal set of positive abnormality assumptions that is consistent with the observations. (Ab's not in the diagnosis are assumed to be false.)

Diagnosis

Ab(a)	Ab(b)	entailed?	consistent?
0	0	no	no
1	0	no	yes
0	1	no	yes
1	1	yes	yes

- Abductive diagnoses:
  { Ab(a), Ab(b) }
- Consistency-based diagnoses:
  { Ab(a) } and { Ab(b) }

SAT-Modulo Theories

- Idea: propositions can be arithmetic constraints

$P_1 \equiv (a < b)$

$P_2 \equiv (b \leq c)$

$P_3 \equiv (c < a)$

Eager Direct: pre-compute and add clauses that capture the constraints between the arithmetic propositions. E.g.:

$(\neg P_1 \vee \neg P_2 \vee \neg P_3) \wedge (P_1 \vee P_2 \vee P_3)$

Eager Circuit: Assume that numeric variables can be represented by k bits for some fixed k. Add clauses that represent arithmetic circuit for each proposition.

E.g., use 2-bit numbers, a is (A_1, A_0), b is (B_1, B_0):

$P_1 \equiv (B_1 \wedge \neg A_1) \vee ((B_1 \equiv A_1) \wedge (B_0 \wedge \neg A_0))$

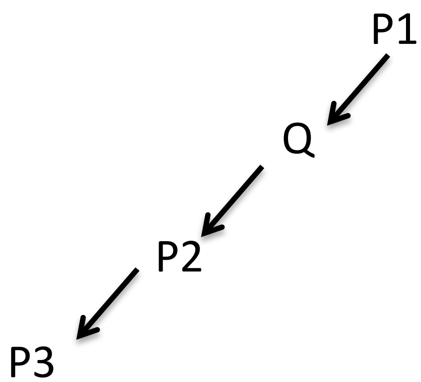
SAT-Modulo Theories

$P_1 \equiv (a < b)$

$P_2 \equiv (b \leq c)$

$P_3 \equiv (c < a)$

Lazy: Do not pre-compute any new clauses. Run DPLL. Whenever an arithmetic proposition is made true or false, check if the set of all such propositions is consistent, using an external solver. If inconsistent, backtrack. Use clause learning to record the reason for backtracking.



Inconsistent! Learn (~P1 v ~P2 v ~P3)

Approximate Inference

- One technique for approximate inference is to compute upper and lower bounds on a theory, where the bounds are in a restricted subset of logic that is tractable
- Horn bounds:
  - There is a unique Horn LUB, equivalent to all the Horn clauses entailed by the theory
  - There can be many Horn GLBs, each is a weakest set of Horn clauses that entails the theory

$$L \in GLB(T)$$

$$U = LUB(T)$$

$$L \models T \models U$$

Approximate Inference

$T = \{(P \vee Q), (\neg P \vee \neg R \vee S), (\neg Q \vee \neg R \vee S), (\neg S \vee \neg A)\}$

To compute LUB, add all resolvants; eliminate non-Horn clauses; eliminate clauses entailed by the other clauses

Resolvants:

$(Q \vee \neg R \vee S)$

$(P \vee \neg R \vee S)$

$(\neg R \vee S)$

$(\neg R \vee \neg A)$

Entailed Horn clauses are:

$\{(\neg P \vee \neg R \vee S), (\neg Q \vee \neg R \vee S), (\neg S \vee \neg A), (\neg R \vee S), (\neg R \vee \neg A)\}$

Removing clauses entailed by other clauses gives LUB:

$\{(\neg S \vee \neg A), (\neg R \vee S)\}$

Approximate Inference

$T = \{(P \vee Q), (\neg P \vee \neg R \vee S), (\neg Q \vee \neg R \vee S), (\neg S \vee \neg A)\}$

To compute GLBs, try all ways of strengthening each non-Horn clause to Horn by removing literals from it. Simplify the resulting set of clauses.

First way to strengthen the one non-Horn clause:

$\{(P), (\neg P \vee \neg R \vee S), (\neg Q \vee \neg R \vee S), (\neg S \vee \neg A)\}$

Simplifying gives GLB_1:

$\{(P), (\neg R \vee S), (\neg S \vee \neg A)\}$

Second way to strengthen the one non-Horn clause:

$\{(Q), (\neg P \vee \neg R \vee S), (\neg Q \vee \neg R \vee S), (\neg S \vee \neg A)\}$

Simplifying gives GLB_2:

$\{(Q), (\neg R \vee S), (\neg S \vee \neg A)\}$

Approximate Inference

- Answering query F using bounds:
  - If F is Horn, then T|=F iff LUB|=F
  - Else: if LUB|=F, then T|=F
  - Else: if for *all* GLB L, L|=/=F, then T|=/=F
  - Else: we cannot tell if query is entailed by original theory by using the bounds

$T = \{(P \vee Q), (\neg P \vee \neg R \vee S), (\neg Q \vee \neg R \vee S), (\neg S \vee \neg A)\}$

LUB:

$\{(\neg S \vee \neg A), (\neg R \vee S)\}$

GLB_1:

$\{(P), (\neg R \vee S), (\neg S \vee \neg A)\}$

GLB_2:

$\{(Q), (\neg R \vee S), (\neg S \vee \neg A)\}$

Query (S)?

No, because S is Horn, and LUB|=/= S

Query $(P \vee \neg R \vee S)$?

Yes, because LUB |=$(P \vee \neg R \vee S)$

Query $(P \vee S)$?

Unknown, because LUB |=/=$(P \vee S)$ and GLB_1 |=$(P \vee S)$

Query $(R \vee S)$?

No, because it is not entailed by the LUB or any of the GLBs

Multiple Agents

- "Modal logic" extends FOL by including predicates whose arguments are formulas rather than terms
- We can use it to represent the beliefs of different agents

$B_A(P \vee Q)$

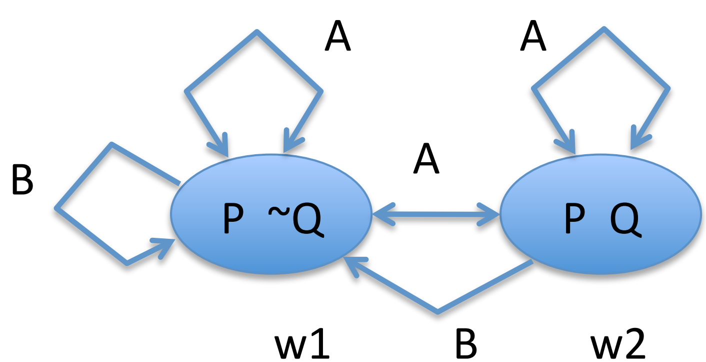
Agent A believes $(P \vee Q)$

$B_A(P) \vee B_A(Q)$

Agent A believes P, or Agent A believes Q

Multiple Agents

- The semantics of modal logic is based on "structures": a set of possible worlds, and a reachability relationship between worlds
- An agent A believes P in world *w* iff P is true in all worlds *w′* reachable from *w* by the A relation



In w1

A believes P?	YES
A believes ~Q?	NO
A believes B believes ~Q?	YES

Bayesian Networks

- A Bayesian network can be encoded in logic by introducing propositions that represent independent random (biased) coin flips
- The probability of a model is the probability of the particular set of coin flips in the model
- By weighting models by their probability, probabilistic inference becomes weighted model counting

Limits of FOL

- FOL is the strongest logic with a complete proof theory
- However, many commonplace mathematical notions cannot be expressed in FOL
- For example, we cannot write a formula that says "P(x,y) is precisely the transitive closure of Q(x,y)" if Q is infinite
- It can be useful in practice to use logics that are more expressive than FOL, even if they do not have complete proof theories
  - In practice, you give up a proof when you run out of time, even if the proof theory is complete.

Gödel

- But things are even worse for logic than the limits of its expressivity
- There are mathematical theorems that can be expressed in FOL, and that are demonstratively true, but for which no FOL proof exists.
  - Because FOL is complete, this means they are true but not entailed by the semantics of FOL either
- That is Gödel's famous construction

Limits of Logic

- Since FOL is the strongest complete logic, this means that there is no way around the limitation by finding a "stronger" logic
- Another general limitation of logic discovered by Gödel is that any logic that is strong enough to be able to prove its own consistency must be inconsistent